Lecture slides for

Automated Planning: Theory and Practice

Chapter 9
Heuristics in Planning

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Planning as Nondeterministic Search

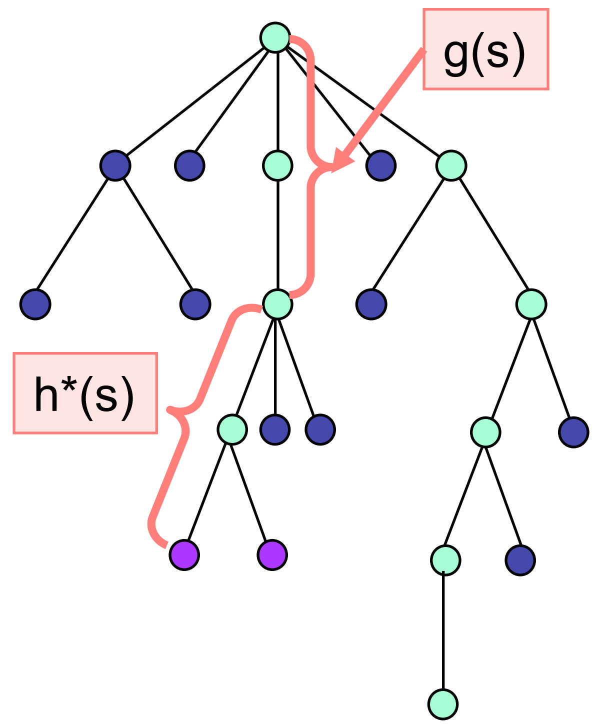
Abstract-search(u)
    if Terminal(u) then return(u)
    $u \leftarrow$ Refine(u)   ;; *refinement step*
    $B \leftarrow$ Branch(u)   ;; *branching step*
    $B' \leftarrow$ Prune(B)   ;; *pruning step*
    if $B' = \emptyset$ then return(failure)
    $\boxed{\text{nondeterministically choose } v \in B'}$
    return(Abstract-search(v))
end

Making it Deterministic

```
Depth-first-search(u)
    if Terminal(u) then return(u)
    u ← Refine(u)              ;;  refinement step
    B ← Branch(u)              ;;  branching step
    C ← Prune(B)               ;;  pruning step
    while C ≠ ∅ do
        v ← Select(C)          ;;  node-selection step
        C ← C − {v}
        π ← Depth-first-search(v)
        if π ≠ failure then return(π)
    return(failure)
end
```

Node-Selection Heuristic

- Suppose we're searching a **tree** in which each edge (s,s') has a cost $c(s,s')$
  - If p is a path, let $c(p)$ = sum of the edge costs
  - For classical planning, this is the length of p



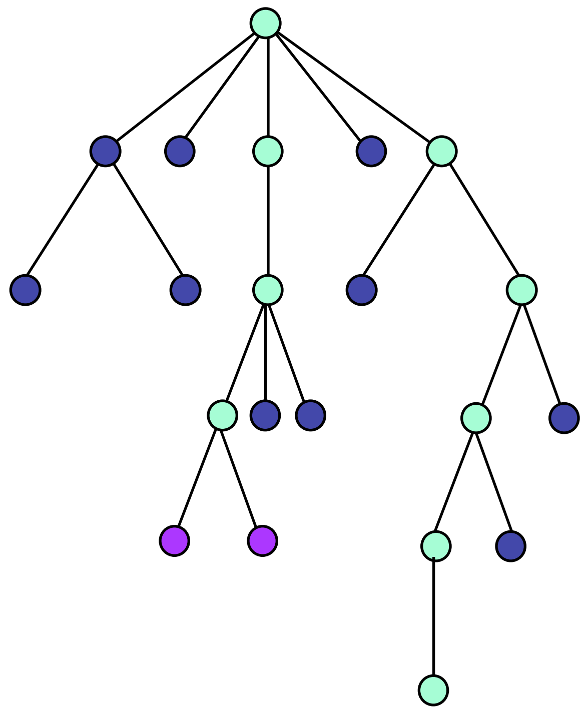
- For every state s, let
  - $g(s)$ = cost of the path from s_0 to s
  - $h^*(s)$ = least cost of all paths from s to goal nodes
  - $f^*(s) = g(s) + h^*(s)$ = least cost of all paths from s_0 to goal nodes that go through s

- Suppose $h(s)$ is an estimate of $h^*(s)$
  - Let $f(s) = g(s) + h(s)$
    - » $f(s)$ is an estimate of $f^*(s)$
  - h is *admissible* if for every state s, $0 \le h(s) \le h^*(s)$
  - If h is admissible then f is a lower bound on f^*

The A* Algorithm

- A* on trees:

  loop
  - choose the leaf node s such that $f(s)$ is smallest
  - if s is a solution then return it and exit
  - expand it (generate its children)



- On graphs, A* is more complicated
  - additional machinery to deal with multiple paths to the same node
- If a solution exists (and certain other conditions are satisfied), then:
  - If $h(s)$ is admissible, then A* is guaranteed to find an optimal solution
  - The more "informed" the heuristic is (i.e., the closer it is to h^*), the smaller the number of nodes A* expands
  - If $h(s)$ is within c of being admissible, then A* is guaranteed to find a solution that's within c of optimal

Heuristic Functions for Planning

- $\Delta^*(s,p)$: minimum distance from state s to a state that contains p
- $\Delta^*(s,s')$: minimum distance from state s to a state that contains all of the literals in s'
  - Hence $h^*(s) = \Delta^*(s,g)$ is the minimum distance from s to the goal
- For $i = 0, 1, 2, \ldots$ we will define the following functions:
  - $\Delta_i(s,p)$: an estimate of $\Delta^*(s,p)$
  - $\Delta_i(s,s')$: an estimate of $\Delta^*(s,s')$
  - $h_i(s) = \Delta_i(s,g)$, where g is the goal

Heuristic Functions for Planning

- $\Delta_0(s,s')$ = what we get if we pretend that
  - Negative preconditions and effects don't exist
  - The cost of achieving a set of preconditions $\{p_1, \ldots, p_n\}$ is the sum of the costs of achieving each p_i separately

$$\Delta_0(s, p) = \begin{cases} 0, & \text{if } p \in s \\ \infty, & \text{if } p \notin s \text{ and } \forall a \in A,\ p \notin \text{effects}^+(a) \\ \min_a \{1 + \Delta_0(s, \text{precond}^+(a)) \mid p \in \text{effects}^+(a), & \text{otherwise} \end{cases}$$

$$\Delta_0(s, g) = \begin{cases} 0, & \text{if } g \subseteq s, \\ \sum_{p \in g} \Delta_1(s,p), & \text{otherwise} \end{cases}$$

- $\Delta_0(s,s')$ is not admissible, but we don't necessarily care
- Usually we'll want to do a depth-first search, not an A* search
  - This already sacrifices admissibility

Computing Δ_0

- Given s, can compute $\Delta_0(s,p)$ for every proposition p
  - Forward search from s
  - U is a set of sets of propositions

Delta(s)
  for each p do: if $p \in s$ then $\Delta_0(s,p) \leftarrow 0$, else $\Delta_0(s,p) \leftarrow \infty$
  $U \leftarrow \{s\}$
  iterate
    for each a such that $\exists u \in U, \mathrm{precond}(a) \subseteq u$ do
      $U \leftarrow \{u\} \cup \mathrm{effects}^+(a)$
      for each $p \in \mathrm{effects}^+(a)$ do
        $\Delta_0(s,p) \leftarrow \min\{\Delta_0(s,p)\ ,\ 1 + \sum_{q \in \mathrm{precond}(a)} \Delta_0(s,q)\}$
  until no change occurs in the above updates
end

- From this, can compute $h_0(s) = \Delta_0(s,g) = \sum_{p \in g} \Delta_0(s,p)$

Heuristic Forward Search

```
Heuristic-forward-search(π, s, g, A)
    if s satisfies g then return π
    options ← {a ∈ A | a applicable to s}
    for each a ∈ options do Delta(γ(s, a))
    while options ≠ ∅ do
        a ← argmin{Δ0(γ(s, a), g) | a ∈ options}
        options ← options − {a}
        π' ← Heuristic-forward-search(π.a, γ(s, a), g, A)
        if π' ≠ failure then return(π')
    return(failure)
end
```

- This is depth-first search, so admissibility is irrelevant
- This is roughly how the HSP planner works
    - First successful use of an A*-style heuristic in classical planning

Heuristic Backward Search

- HSP can also search backward

```
Backward-search(π, s_0, g, A)
    if s_0 satisfies g then return(π)
    options ← {a ∈ A | a relevant for g}
    while options ≠ ∅ do
        a ← argmin{Δ_0(s_0, γ^{-1}(g, a)) | a ∈ options}
        options ← options − {a}
        π' ← Backward-search(a.π, s_0, γ^{-1}(g, a), A)
        if π' ≠ failure then return(π')
    return failure
end
```

An Admissible Heuristic

$$\Delta_1(s, p) = \begin{cases} 0, & \text{if } p \in s \\ \infty, & \text{if } p \notin s \text{ and } \forall a \in A,\ p \notin \text{effects}^+(a) \\ \min_a \{1 + \Delta_1(s, \text{precond}^+(a)) \mid p \in \text{effects}^+(a), & \text{otherwise} \end{cases}$$

$$\Delta_1(s, g) = \begin{cases} 0, & \text{if } g \subseteq s, \\ \max_{p \in g} \Delta_1(s,p), & \text{otherwise} \end{cases}$$

- $\Delta_1(s, s')$ = what we get if we pretend that
  - Negative preconditions and effects don't exist
  - The cost of achieving a set of preconditions $\{p_1, \ldots, p_n\}$ is the max of the costs of achieving each p_i separately
- This heuristic is admissible; thus it could be used with A*
  - It is not very informed

A More Informed Heuristic

- Δ_2: instead of computing the minimum distance to each p in g, compute the minimum distance to each pair $\{p,q\}$ in g:
  - Analogy to GraphPlan's mutex conditions

$$\Delta_2(s, p) = \begin{cases} 0, & \text{if } p \in s \\ \infty, & \text{if } p \notin s \text{ and } \forall a \in A,\ p \notin \text{effects}^+(a) \\ \min_a \{1 + \Delta_2(s, \text{precond}^+(a)) \mid p \in \text{effects}^+(a), & \text{otherwise} \end{cases}$$

$$\Delta_2(s, \{p,q\}) = \min \begin{Bmatrix} \min_a \{1 + \Delta_2(s, \text{precond}^+(a)) \mid \{p,q\} \subseteq \text{effects}^+(a)\} \\ \min_a \{1 + \Delta_2(s, \{q\} \cup \text{precond}^+(a)) \mid p \in \text{effects}^+(a)\} \\ \min_a \{1 + \Delta_2(s, \{p\} \cup \text{precond}^+(a)) \mid q \in \text{effects}^+(a)\} \end{Bmatrix}$$

$$\Delta_2(s, g) = \begin{cases} 0, & \text{if } g \subseteq s, \\ \max_{p,q} \ \Delta_2(s, \{p,q\}) \mid \{p,q\} \subseteq g\}, & \text{otherwise} \end{cases}$$

More Generally, …

Recall that $\Delta^\star(s,g)$ is the true minimal distance from a state s to a goal g. $\Delta^\star$ can be computed (albeit at great computational cost) according to the following equations:

$$\Delta^\star(s,g) = \begin{cases} 0 & \text{if } g \subseteq s, \\ \infty & \text{if } \forall a \in A, a \text{ is not relevant for } g, \text{ and} \\ \min_a\{1 + \Delta^\star(s, \gamma^{-1}(g,a)) \mid a \text{ relevant for } g\} \\ & \text{otherwise.} \end{cases} \tag{9.4}$$

- From this, can define $\Delta_k(s,g)$ = max distance to each k-tuple $\{p_1,p_2,\ldots,p_k\}$ in g
  - Analogy to k-ary mutex conditions

$$\Delta_k(s,g) = \begin{cases} 0 & \text{if } g \subseteq s, \\ \infty & \text{if } \forall a \in A, a \text{ is not relevant for } g, \\ \min_a\{1 + \Delta^\star(s, \gamma^{-1}(g,a)) \mid a \text{ relevant for } g\} \\ & \text{if } |g| \leq k, \\ \max_{g'}\{\Delta_k(s,g') \mid g' \subseteq g \text{ and } |g'| = k\} \\ & \text{otherwise.} \end{cases} \tag{9.5}$$

$$\Delta_2(s,p) = \begin{cases} 0, & \text{if } p \in s \\ \infty, & \text{if } p \notin s \text{ and } \forall a \in A,\ p \notin \text{effects}^+(a) \\ \min_a \{1 + \Delta_2(s, \text{precond}^+(a)) \mid p \in \text{effects}^+(a), & \text{otherwise} \end{cases}$$

$$\Delta_2(s, \{p,q\}) = \min \begin{Bmatrix} \min_a \{1 + \Delta_2(s, \text{precond}^+(a)) \mid \{p,q\} \subseteq \text{effects}^+(a)\} \\ \min_a \{1 + \Delta_2(s, \{q\} \cup \text{precond}^+(a)) \mid p \in \text{effects}^+(a)\} \\ \min_a \{1 + \Delta_2(s, \{p\} \cup \text{precond}^+(a)) \mid q \in \text{effects}^+(a)\} \end{Bmatrix}$$

$$\Delta_2(s,g) = \begin{cases} 0, & \text{if } g \subseteq s, \\ \max_{p,q}\ \Delta_2(s, \{p,q\}) \mid \{p,q\} \subseteq g\}, & \text{otherwise} \end{cases}$$

$$\Delta_k(s,g) = \begin{cases} 0 & \text{if } g \subseteq s, \\ \infty & \text{if } \forall a \in A, a \text{ is not relevant for } g, \\ \min_a\{1 + \Delta^\star(s, \gamma^{-1}(g,a)) \mid a \text{ relevant for } g\} \\ & \text{if } |g| \leq k, \\ \max_{g'}\{\Delta_k(s,g') \mid g' \subseteq g \text{ and } |g'| = k\} \\ & \text{otherwise.} \end{cases} \tag{9.5}$$

Complexity of Computing the Heuristic

- Takes time $\Omega(n^k)$
- If $k \geq \max(|g|, \max\{|\text{precond}(a)| : a \text{ is an action}\})$ then computing $\Delta(s,g)$ is as hard as solving the entire planning problem

Getting Heuristic Values from a Planning Graph

- Recall how GraphPlan works:

loop

Graph expansion: this takes polynomial time

extend a "planning graph" forward from the initial state until we have achieved a necessary (but insufficient) condition for plan existence

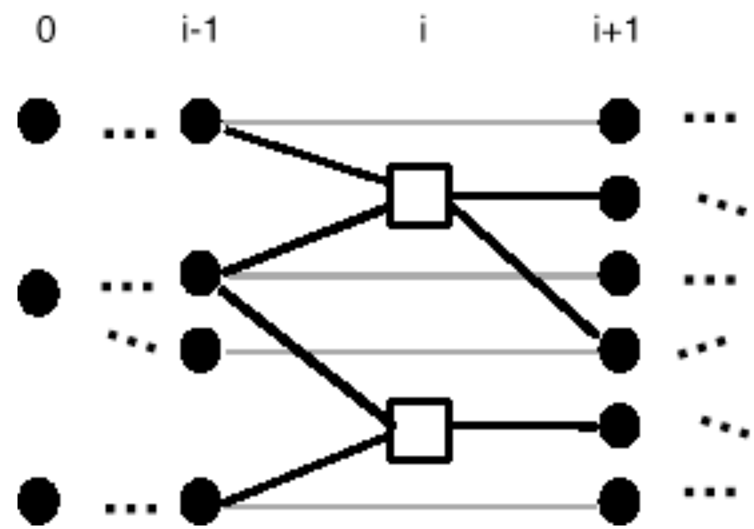
Solution extraction: this takes exponential time

search backward from the goal, looking for a correct plan

if we find one, then return it

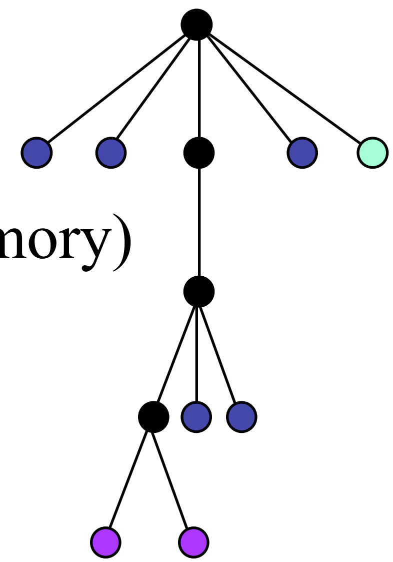
repeat

Using Planning Graphs to Compute *h(s)*



- In the graph, there are alternating layers of ground literals and actions
- The number of "action" layers is a lower bound on the number of actions in the plan
- Construct a planning graph, starting at s
- $\Delta^g(s,p)$ = level of the first layer that "possibly achieves" p
- $\Delta^g(s,g)$ is very close to $\Delta_2(s,g)$
  - $\Delta_2(s,g)$ counts each action individually
  - $\Delta^g(s,g)$ groups together the independent actions in a layer

The FastForward Planner



- Use a heuristic function similar to $h(s) = \Delta^g(s,g)$
  - Some ways to improve it (I'll skip the details)
- Don't want an A*-style search (takes too much memory)
- Instead, use a greedy procedure:

  until we have a solution, do
      expand the current state *s*
      s := the child of *s* for which *h(s)* is smallest
        (i.e., the child we think is closest to a solution)

- There are some ways to improve this (I'll skip the details)
- Can't guarantee how fast it will find a solution, or how good a solution it will find
  - However, it works pretty well on many problems

AIPS-2000 Planning Competition

- FastForward did quite well
- In the this competition, all of the planning problems were classical problems
- Two tracks:
  - "Fully automated" and "hand-tailored" planners
  - FastForward participated in the fully automated track
    - » It got one of the two "outstanding performance" awards
  - Large variance in how close its plans were to optimal
    - » However, it found them very fast compared with the other fully-automated planners

2002 International Planning Competition

- Among the automated planners, FastForward was roughly in the middle
- LPG (graphplan + local search) did much better, and got a "distinguished performance of the first order" award

- It's interesting to see how FastForward did in problems that went beyond classical planning
      » Numbers, optimization
- Example: Satellite domain, numeric version
  - A domain inspired by the Hubble space telescope (a lot simpler than the real domain, of course)
      » A satellite needs to take observations of stars
      » Gather as much data as possible before running out of fuel
  - Any amount of data gathered is a solution
      » Thus, FastForward always returned the null plan

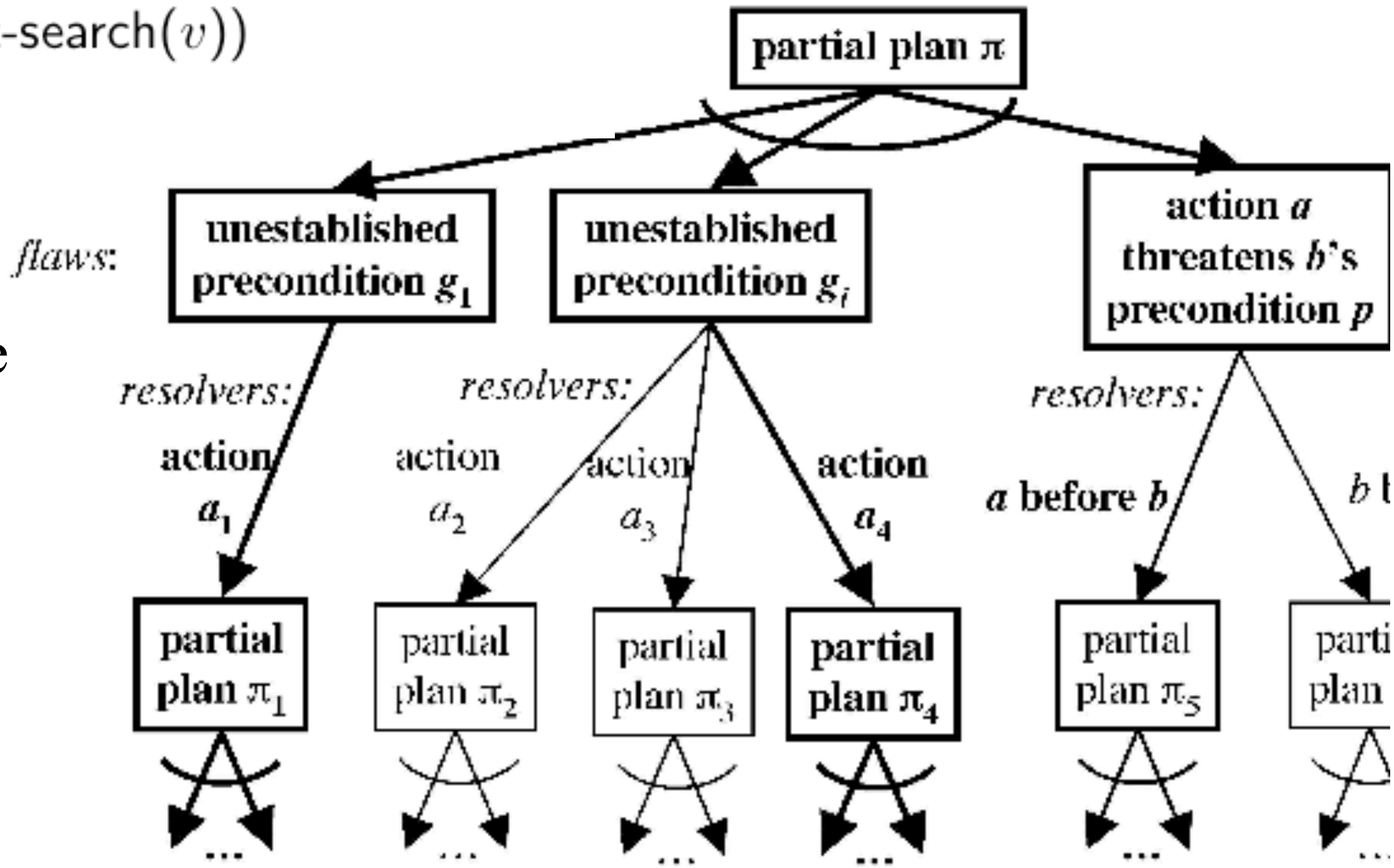
2004 International Planning Competition

- FastForward's author was one of the competition chairs
  - Thus FastForward did not participate

Heuristics for Plan-Space Planning

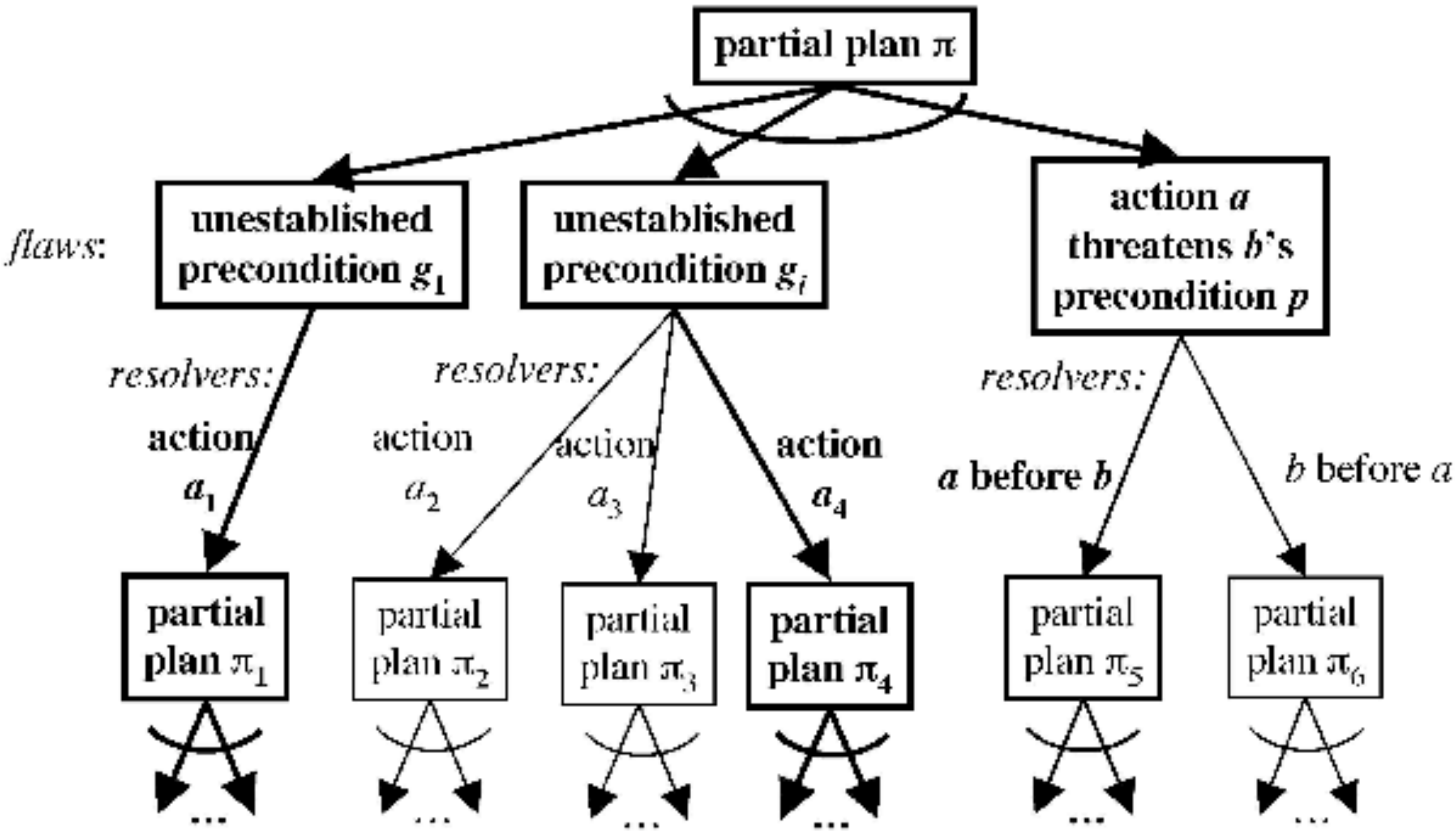
```
Abstract-search(u)
    if Terminal(u) then return(u)
    u ← Refine(u)          ;;  refinement step
    B ← Branch(u)          ;;  branching step
    B′ ← Prune(B)          ;;  pruning step
    if B′ = ∅ then return(failure)
    nondeterministically choose v ∈ B′
    return(Abstract-search(v))
end
```

- For plan-space planning, *refinement* = selecting the next flaw to work on



One Possible Heuristic

- Fewest Alternatives First (FAF)

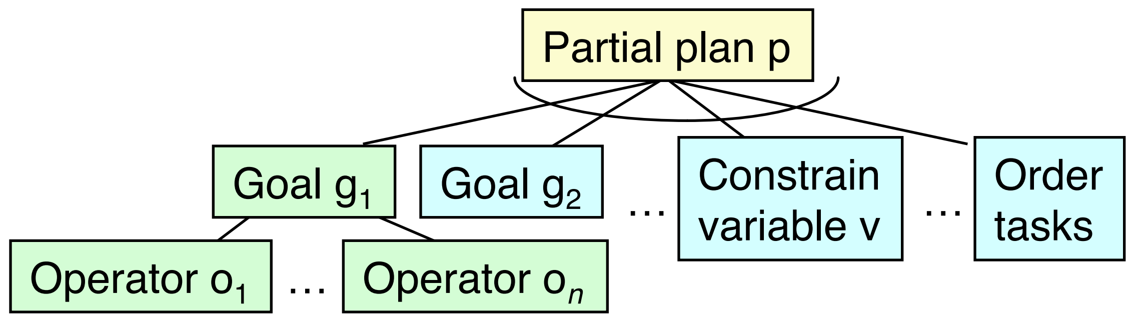


Do Others Work Better?

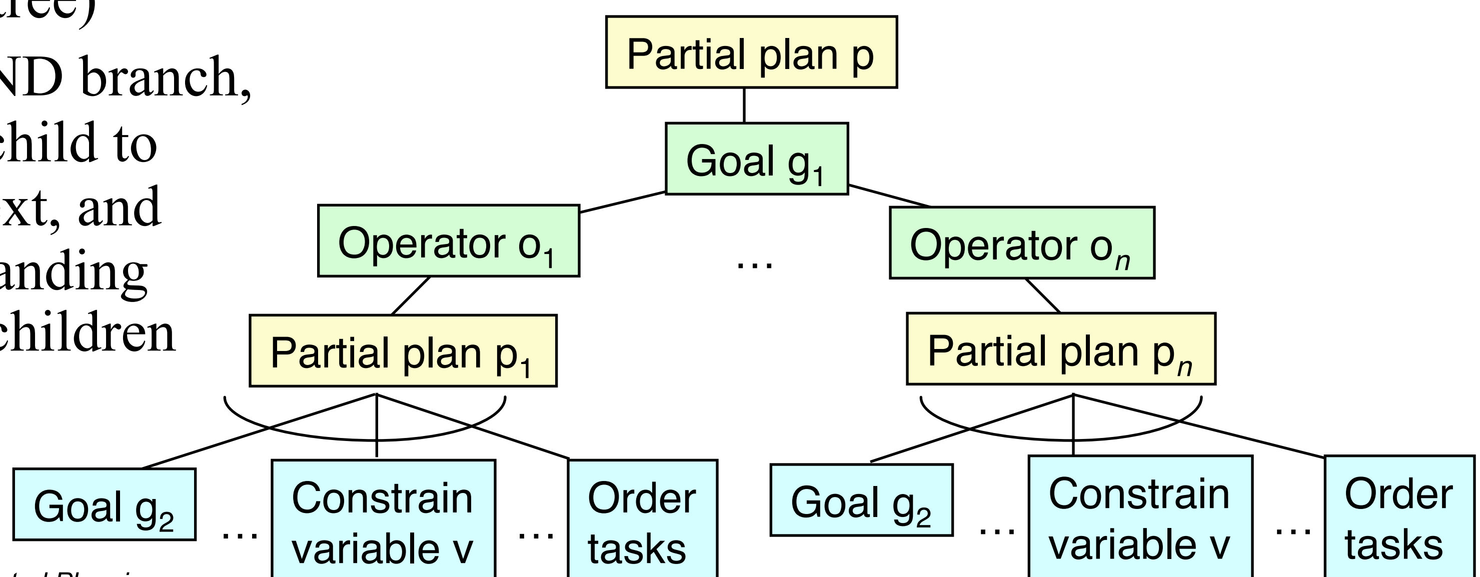
- Sometimes yes, sometimes no
- Limits to how good *any* flaw-selection heuristic can do

Serializing and AND/OR Tree

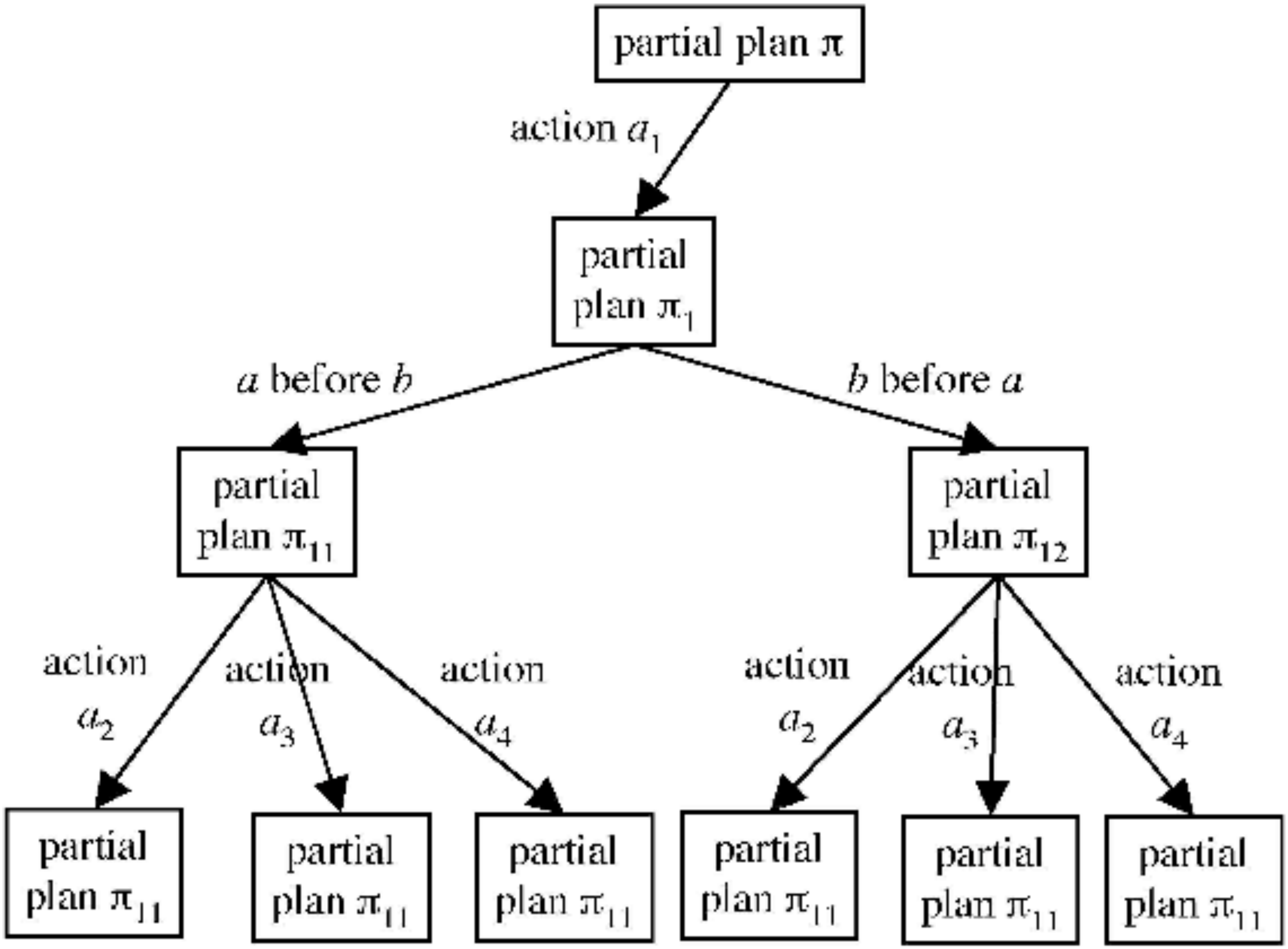
- The search space is an AND/OR tree



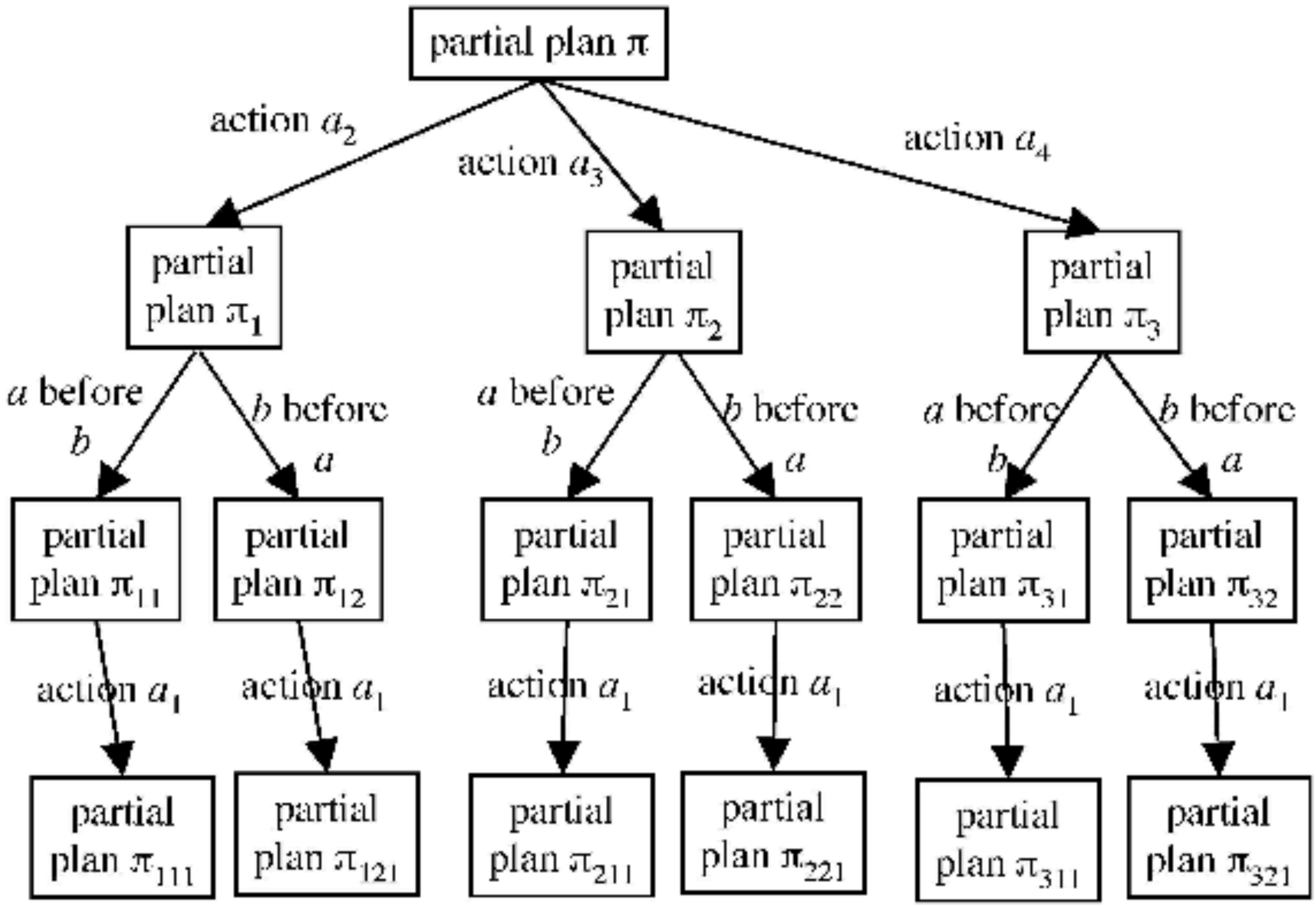
- Deciding what flaw to work on next = *serializing* this tree (turning it into a state-space tree)
  - at each AND branch, choose a child to expand next, and delay expanding the other children



One Serialization



Another Serialization



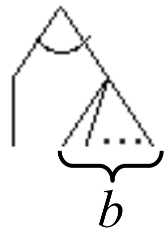
Why Does This Matter?

- Different refinement strategies produce different serializations
  - the search spaces have different numbers of nodes
- In the worst case, the planner will search the entire serialized search space
- The smaller the serialization, the more likely that the planner will be efficient

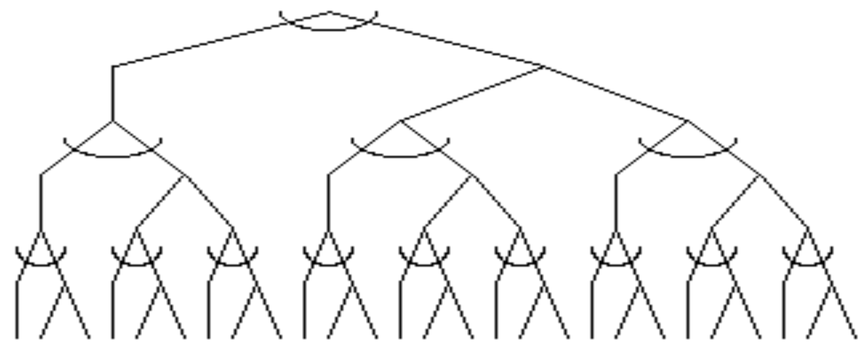
- One pretty good heuristic: fewest alternatives first

How Much Difference Can the Refinement Strategy Make?

- Case study: build an AND/OR graph from repeated occurrences of this pattern:

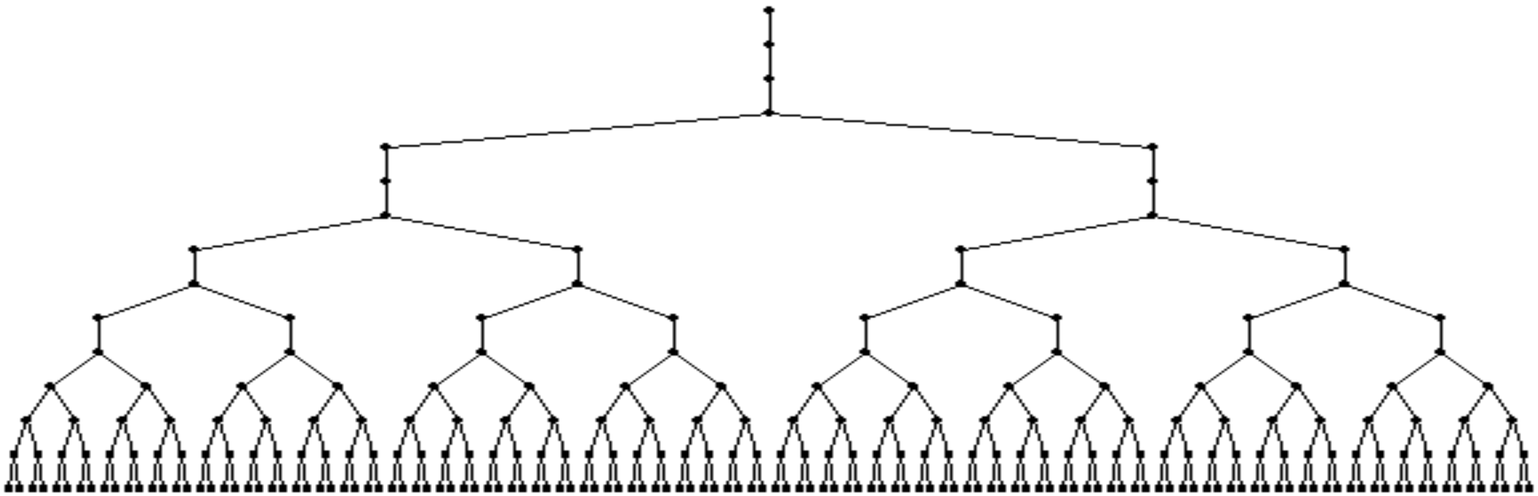


- Example:
  - number of levels $k = 3$
  - branching factor $b = 2$



- Analysis:
  - Total number of nodes in the AND/OR graph is $n = \Theta(b^k)$
  - How many nodes in the best and worst serializations?

Case Study, Continued



- The best serialization contains $\Theta(b^{2^k})$ nodes
- The worst serialization contains $\Theta(2^k b^{2^k})$ nodes
  - The size differs by an exponential factor
  - But both serializations are *doubly* exponentially large
- To do better, need good node selection, branching, pruning