

Intelligent Agents

Inference in First Order Logic

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Remember ...

... in the last lecture we started to introduce *resolution*.

- Resolution calculus is a basic approach for performing logical proofs on a machine.
- Logical formula must be rewritten into **clause form**, using **equivalence rules**.
- To perform a **resolution step** on a pair of clauses, literals must be **unified**.

Outline

- Clausal Form
- Substitution and Unification
- Proofs by Resolution
- Prolog and SLD-Resolution
- Applications of Resolution
- Deductive Planning
 - Situation Calculus
 - Frame Problem
- SAT-Planning

Clause Form

- **Conjunctive Normal Form (CNF):**
Conjunction of disjunctions of literals

$$\bigwedge_{i=1}^n (\bigvee_{j=1}^m L_{ij})$$

- **Clause Form:**
Set of disjunctions of literals (can be generated from CNF)

Rewriting of formulas to clause form:

8 steps, illustrated with example

$$\forall x [B(x) \rightarrow (\exists y [O(x, y) \wedge \neg P(y)] \wedge \neg \exists y [O(x, y) \wedge O(y, x)] \wedge \forall y [\neg B(y) \rightarrow \neg E(x, y)])]$$

Clause Form cont.

(0) Original Formula

$$\forall x [B(x) \rightarrow (\exists y [O(x, y) \wedge \neg P(y)] \wedge \neg \exists y [O(x, y) \wedge O(y, x)] \wedge \forall y [\neg B(y) \rightarrow \neg E(x, y)])]]$$

(1) Remove Implications

$$\forall x [\neg B(x) \vee (\exists y [O(x, y) \wedge \neg P(y)] \wedge \neg \exists y [O(x, y) \wedge O(y, x)] \wedge \forall y [\neg(\neg B(y)) \vee \neg E(x, y)])]]$$

(2) Reduce scopes of negation

$$\forall x [\neg B(x) \vee (\exists y [O(x, y) \wedge \neg P(y)] \wedge \forall y [\neg O(x, y) \vee \neg O(y, x)] \wedge \forall y [B(y) \vee \neg E(x, y)])]]$$

(3) Skolemization (remove existential quantifiers)

Replace existentially quantified variables by constant/function symbols.

$\exists x p(x)$ becomes $p(C)$

“There exists a human who is a student.” is satisfiable if there exists a constant in the universe \mathcal{U} for which the sentence is true.

“Human C is a student.” is satisfiable if the constant symbol C can be interpreted such that relation p is true.)

Clause Form cont.

Skolemization cont.

If an existentially quantified variable is in the scope of a universally quantified variable, it is replaced by a function symbol dependent of this variable:

$$\forall x \exists y p(x) \wedge q(x, y) \text{ becomes } \forall x p(x) \wedge q(x, f(x))$$

(“For all x holds, x is a positive integer and there exists a y which is greater than x .” is satisfiable if for each x exists an y such that the relation “greater than” holds. E.g., $f(x)$ can be interpreted as successor-function.)

*Skolemization is **no equivalence transformation**. A formula and its Skolemization are only equivalent with respect to satisfiability!
The skolemized formula has a model iff the original formula has a model.*

$$\forall x [\neg B(x) \vee ((O(x, f(x)) \wedge \neg P(f(x))) \wedge \forall y [\neg O(x, y) \vee \neg O(y, x)] \wedge \forall y [B(y) \vee \neg E(x, y)])]$$

Clause Form cont.

(4) Standardize variables (“bounded renaming”)

A variable bound by a quantifier is a “dummy” and can be renamed. Provide that each variable of universal quantifier has a different name. (Problematic case: free variables)

$$\forall x[\neg B(x) \vee ((O(x, f(x)) \wedge \neg P(f(x))) \wedge \forall y[\neg O(x, y) \vee \neg O(y, x)] \wedge \forall z[B(z) \vee \neg E(x, z)])]$$

(5) Prenex-form

Move universal quantifiers to front of the formula.

$$\forall x \forall y \forall z [\neg B(x) \vee ((O(x, f(x)) \wedge \neg P(f(x))) \wedge (\neg O(x, y) \vee \neg O(y, x)) \wedge (B(z) \vee \neg E(x, z)))]$$

(6) CNF

(Repeatedly apply the distributive laws)

$$\forall x \forall y \forall z [(\neg B(x) \vee O(x, f(x))) \wedge (\neg B(x) \vee \neg P(f(x))) \wedge (\neg B(x) \vee \neg O(x, y) \vee \neg O(y, x)) \wedge (\neg B(x) \vee B(z) \vee \neg E(x, z))]$$

Clause Form cont.

(7) Eliminate Conjunctions

If necessary, rename variable such that each disjunction has a different set of variables.

The truth of a conjunction entails that all its parts are true.

$$\forall x[\neg B(x) \vee O(x, f(x))], \forall w[\neg B(w) \vee \neg P(f(w))], \forall u \forall y[\neg B(u) \vee \neg O(u, y) \vee \neg O(y, u)], \forall v \forall z[\neg B(v) \vee B(z) \vee \neg E(v, z)]$$

(8) Eliminate Universal Quantifiers

Clauses are implicitly universally quantified.

$M =$

$$\{\neg B(x) \vee O(x, f(x)), \neg B(w) \vee \neg P(f(w)), \neg B(u) \vee \neg O(u, y) \vee \neg O(y, u), \neg B(v) \vee B(z) \vee \neg E(v, z)\}$$

Substitution

- A **substitution** is a set

$$\theta = \{v_1 \leftarrow t_1, \dots, v_n \leftarrow t_n\}$$

of replacements of **variables** v_i by **terms** t_i .

- If θ is a substitution and E an expression, $E' = E\theta$ is called **instance** of E .

E' was derived from E by applying θ to E .

Example

- $E = p(x) \vee (\neg q(x, y) \wedge p(f(x)))$
- $\theta = \{x \leftarrow C\}$
- $E\theta = p(C) \vee (\neg q(C, y) \wedge p(f(C)))$
- Special case: alphabetic substitution (variable renaming).

Composition of Substitutions

- Let be

$$\theta = \{u_1 \leftarrow t_1, \dots, u_n \leftarrow t_n, v_1 \leftarrow s_1, \dots, v_k \leftarrow s_k\} \text{ and}$$

$$\sigma = \{v_1 \leftarrow r_1, \dots, v_k \leftarrow r_k, w_1 \leftarrow q_1, \dots, w_m \leftarrow q_m\}.$$

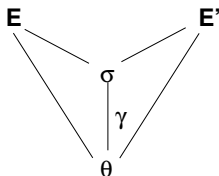
- The composition is defined as

$$\theta\sigma =_{Def} \{u_1 \leftarrow t_1\sigma, \dots, u_n \leftarrow t_n\sigma, v_1 \leftarrow s_1\sigma, \dots, v_k \leftarrow s_k\sigma, w_1 \leftarrow q_1, \dots, w_m \leftarrow q_m\}$$

- Composition of substitutions is not commutative!

Unification

- Let be $\{E_1 \dots E_n\}$ a set of expressions. A substitution θ is a **unificator** of $E_1 \dots E_n$, if $E_1\theta = E_2\theta \dots = E_n\theta$.
- A unificator θ is called **most general unifier** (mgu), if for each other unificator σ for $E_1 \dots E_n$ there exists a substitution γ with $\sigma = \theta\gamma$.
- Theorem: If a unificator exists, then also an mgu exists.



There are lots of unification algorithms, e.g. one proposed by Robinson.

Examples

- | | | |
|-----|--|--|
| (1) | $\{P(x), P(A)\}$ | $\theta = \{x \leftarrow A\}$ |
| (2) | $\{P(f(x), y, g(y)), P(f(x), z, g(x))\}$ | $\theta = \{y \leftarrow x, z \leftarrow x\}$ |
| (3) | $\{P(f(x, g(A, y)), g(A, y)), P(f(x, z), z)\}$ | $\theta = \{z \leftarrow g(A, y)\}$ |
| (4) | $\{P(x, f(y), B), P(x, f(B), B)\}$ | $\sigma = \{x \leftarrow A, y \leftarrow B\}$
$\theta = \{y \leftarrow B\}$ |

In (4) holds:

θ is more general than σ : $\sigma = \theta\gamma$, with $\gamma = \{x \leftarrow A\}$

θ is mgu for $\{P(x, f(y), B), P(x, f(B), B)\}$

Unification Algorithm

For a given set of formula S :

- 1 Let be $\theta = \{\}$
 - 2 While $|S| > 1$ DO
 - 1 Calculate the disagreement set D of S
 - 2 If D contains a variable x and a term t in which x does not occur
Then $\theta = \theta\{x \leftarrow t\}$ and $S = S\theta$
Else stop (S not unifiable)
 - 3 Return θ as mgu of S
- Since S is the set of all formula, it has size one if all formula are identical (unified by θ).
 - Calculation of disagreement set see practice

Resolution

A clause

$$C = \bigvee_{i=1}^n L_i$$

can be written as set

$$C = \{L_1, \dots, L_n\}.$$

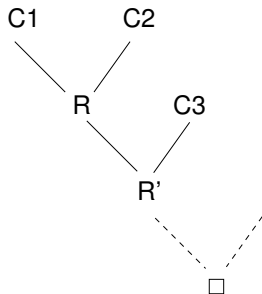
Let be C_1 , C_2 and R clauses. R is called **resolvent** of C_1 and C_2 if:

- There are alphabetical substitutions σ_1 and σ_2 such that $C_1\sigma_1$ and $C_2\sigma_2$ have no common variables.
- There exists a set of literals $L_1, \dots, L_m \in C_1\sigma_1$ ($m \geq 1$) and $L'_1, \dots, L'_n \in C_2\sigma_2$ ($n \geq 1$) such that $L = \{\neg L_1, \neg L_2, \dots, \neg L_m, L'_1, L'_2, \dots, L'_n\}$ are unifiable with θ as mgu of L .
- R has the form:

$$R = ((C_1\sigma_1 \setminus \{L_1, \dots, L_m\}) \cup (C_2\sigma_2 \setminus \{L'_1, \dots, L'_n\}))\theta.$$

Resolution cont.

Derivation of a clause by application of the resolution rule can be described by a **refutation tree**:



Illustration

$$C_1 = \{P(f(x)), \neg Q(z), P(z)\}$$

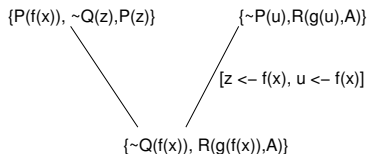
$$C_2 = \{\neg P(x), R(g(x), A)\}$$

$$\sigma_1 = \{\}, \sigma_2 = \{x \leftarrow u\}$$

$$L = \{P(f(x)), P(z), \neg\neg P(x)\} = \{P(f(x)), P(z), P(u)\}$$

$$\theta = \{z \leftarrow f(x), u \leftarrow f(x)\}$$

$$R = [(\{P(f(x)), \neg Q(z), P(z)\} \setminus \{P(f(x)), P(z)\}) \cup (\{\neg P(u), R(g(u), A)\} \setminus \{P(u)\})] \theta = \{\neg Q(f(x)), R(g(f(x)), A)\}$$



Resolution Proofs

- To prove that formula G (assertion) logically follows from a set of formula (axioms) $F_1 \dots F_n$:

Resolution Proof Strategy

- 1 Include the negated assumption in the set of axioms.
 - 2 Try to derive a contradiction (empty clause).
- Theorem:
A set of clauses is not satisfiable, if the empty clause (\square) can be derived with a resolution proof.
 - (Contradiction:
 $C_1 = A, C_2 = \neg A$, stands for $(A \wedge \neg A)$ and $(A \wedge \neg A) \vdash \square$)

Example

- Axiom: “All humans are mortal” Fact: “Socrates is human”
(Both are non-logical: their truth is presupposed)
- Assertion

“Socrates is mortal.”

- Formalization:

$$F_1 : \forall x : \text{Human}(x) \rightarrow \text{Mortal}(x)$$

$$F_2 : \text{Human}(S)$$

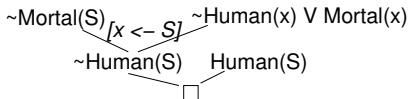
$$F_3 : \neg \text{Mortal}(S) \quad (\text{negation of assertion})$$

- Clause form:

$$F'_1 : \neg \text{Human}(x) \vee \text{Mortal}(x)$$

$$F'_2 : \text{Human}(S)$$

$$F'_3 : \neg \text{Mortal}(S)$$



Soundness and Completeness of Res.

- A calculus is **sound**, if only such conclusions can be derived which also hold in the model.
- A calculus is **complete**, if all conclusions can be derived which hold in the model.
- The resolution calculus is sound and refutation complete.

Refutation completeness means, that if a set of formula (clauses) is unsatisfiable, then resolution will find a contradiction. Resolution cannot be used to generate all logical consequences of a set of formula, but it can establish that a given formula is entailed by the set. Hence, it can be used to find all answers to a given question, using the “negated assumption” method.

Remarks

- The proof ideas will given for resolution for propositional logic (or ground clauses) only.
- For FOL, additionally, a lifting lemma is necessary and the proofs rely on Herbrand structures.
- We cover elementary concepts of logic only.
- For more details, see
 - Ghallab, Nau, & Traverso, Appendix B and chapter 12
 - Uwe Schöning, Logik für Informatiker, 5. Auflage, Spektrum, 2000.
 - Volker Sperschneider & Grigorios Antoniou, Logic – A foundation for computer science, Addison-Wesley, 1991.

Resolution Theorem

Theorem: A set of clauses F is not satisfiable iff the empty clause \square can be derived from F by resolution.

- **Soundness:**

(Proof by contradiction)

Assume that \square can be derived from F . If that is the case, two clauses $C_1 = \{L\}$ and $C_2 = \{\neg L\}$ must be contained in F . Because there exists no model for $L \wedge \neg L$, F is not satisfiable.

- **Refutation completeness:**

(Proof by induction over the number n of atomic formulas in F)

Assume that F is a set of formula which is not satisfiable. Because of the compactness theorem, it is enough to consider the case that a finite non-satisfiable subset of formula exists in F .

To show: \square is derived from F . (see e.g., Schönig)

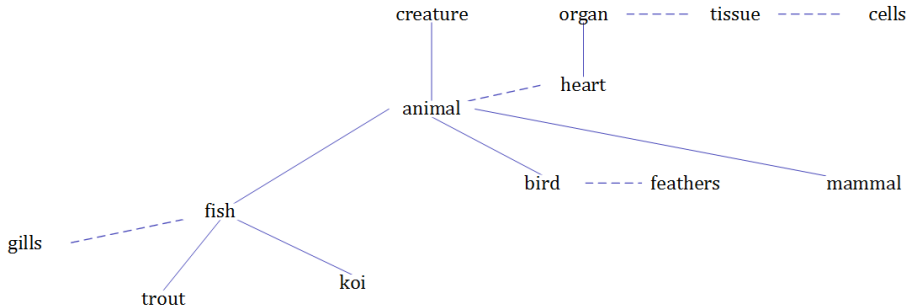
Resolution Strategies

- In general, there are many possibilities, to find two clauses, which are resolvable. Of the many alternatives, there are possibly only a few which help to derive the empty clause \leftrightarrow combinatorial explosion!
- For feasible algorithms: use a resolution **strategy**
- E.g., exploit **subsumption** to keep the knowledge space, and therefore the search space, small.
Remove all sentences which are subsumed (more special than) an existing sentence.
If $P(x)$ is in the knowledge base, sentences as $P(A)$ or $P(A) \vee Q(B)$ can be removed.
- Well known efficient strategy: **SLD-Resolution** (*linear resolution with selection function for definite clauses*) (e.g. used in Prolog)

SLD-Resolution

- **linear**: Use a sequence of clauses ($C_0 \dots C_n$) starting with the negated assertion C_0 and ending with the empty clause C_n . Each C_i is generated as resolvent from C_{i-1} and a clause from the original set of axioms.
- **Selection function** (for the next literal which will be resolved) e.g. top-down-left-to-right in PROLOG; makes the strategy **incomplete!** (“user” must order clauses in a suitable way)
- **definite Horn clauses**: A Horn clause contains maximally one positive literal; a definite Horn clause contains exactly one positive literal (Prolog rule)

Example Prolog Program – Semantic Net



- Has a trout gills?
- Has a fish cells?

Example Prolog Program – Semantic Net

```

/* Example of a hierarchical semantic network in PROLOG */
/* *****/

/* explicit isa and has links */
/* facts */
isa( animal , creature ).
isa( fish , animal ).
isa( trout , fish ).
isa( heart , organ ).
hasprop( animal , heart ).
hasprop( organ , tissue ).
hasprop( tissue , cells ).

/* ----- */
/* Reasoning Rules */

is( A,B ) :- isa( A,B ).          /* Transitivity of is */
is( A,B ) :- isa( A,C ), is( C,B ).

has( A,X ) :- hasprop( A,X ).    /* Transitivity of has */
has( X,Z ) :- hasprop( X,Y ), has( Y,Z ).

has( A,X ) :- hasprop( A,Y ), isa( Y,X ). /* Generalizing has wrt is */

has( A,X ) :- is_a( A,B ), has( B,X ). /* Inheritance of has wrt is */

```

Prolog

	PROLOG	Logic	
Fact	<pre>isa(fish, animal). isa(trout, fish).</pre>	<pre>isa(Fish,Animal) isa(Trout,Fish)</pre>	Positive Literal
Rule	<pre>is(X,Y) :- isa(X,Y). is(X,Z) :- isa(X,Y), is(Y,Z).</pre>	<pre>is(x,y) ∨ ¬isa(x,y) is(x,z) ∨ ¬isa(x,y) ∨ ¬is(y,z)</pre>	Definite Clause
Query	<pre>is(trout, animal). is(fish, X).</pre>	<pre>¬is(Trout, Animal) ¬is(Fish, x)</pre>	Assertion

Prolog

- $:-$ denotes the “reversed” implication arrow.

$$\text{is}(X, Z) \text{ :- isa}(X, Y), \text{is}(Y, Z) .$$

is Prolog for:

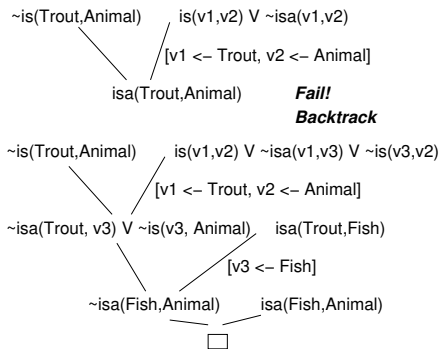
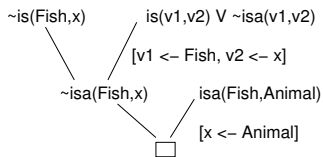
$$\begin{aligned} \text{isa}(x, y) \wedge \text{is}(y, z) \rightarrow \text{is}(x, z) &\equiv \\ \neg(\text{isa}(x, y) \wedge \text{is}(y, z)) \vee \text{is}(x, z) &\equiv \\ \neg\text{isa}(x, y) \vee \neg\text{is}(y, z) \vee \text{is}(x, z) & \end{aligned}$$

- Variables which occur in the head of a clause are implicitly universally quantified. Variables which occur only in the body are existentially quantified.

$$\forall x \forall z \exists y : \neg\text{isa}(x, y) \vee \neg\text{is}(y, z) \vee \text{is}(x, z)$$

Prolog Example

- Query: $\text{is}(\text{fish}, X)$
(stands for $\exists x \text{ is}(\text{Fish}, x)$)
- Negation of query: $\neg \exists x : \text{is}(\text{Fish}, x) \equiv \forall x : \neg \text{is}(\text{Fish}, x)$
- SLD-Resolution: (*extract*)



Remarks on Prolog

- When writing Prolog programs, one should know how the interpreter is working (i.e., understand SLD-resolution)
- Sequence of clauses has influence whether an assertion which follows logically from a set of clauses can be derived!
- **Efficiency**: Facts before rules
- **Termination**: non-recursive rule before recursive.

```
% Program
isa(trout, fish).
isa(fish, animal).
```

```
is(X, Z) :- is(X, Y), isa(Y, Z).
is(X, Y) :- isa(X, Y).
```

```
% Query
? is(trout, animal).
```

```
is(trout, Y), isa(Y, animal)
is(trout, Y'),
isa(Y', animal),
isa(Y, animal)
...
```

Applications of Resolution Calculus

- PROLOG
- as a *basic* method for theorem proving (others: e.g. tableaux)
- Question Answering Systems

- Yes/No-Questions: Assertion/Query *mortal(s)*
- Query *is(trout, X)* corresponds to “What is a trout?”
The variable *X* is instantiated during resolution and the answer is “a fish”.
- *buys(peter, john, X)*: “What does John buy from Peter?”
- *buys(peter, X, car)*: “Who buys a car from Peter?”

Theorem Provers

- Theorem provers typically are more general than Prolog: not only Horn clauses but full FOL; no interleaving of logic and control (i.e. ordering of formulas has no effect on result)
- Examples: Boyer-Moore (1979) theorem prover; OTTER, Isabelle
- Theorem provers for mathematics, for verification of hardware and software, for deductive program synthesis.

Forward- and Backward Chaining

- Rules (e.g. in Prolog) have the form:
Premises \rightarrow *Conclusion*
- All rule-based systems (production systems, planners, inference systems) can be realized using either **forward-chaining** or **backward-chaining** algorithms.
- Forward chaining: Add a new fact to the knowledge base and derive all consequences (data-driven)
- Backward chaining: Start with a goal to be proved, find implication sentences that would allow to conclude the goal, attempt to prove the premises, etc.
- Well known example for a **backward reasoning expert system**:
MYCIN (diagnosis of bacterial infections)

Logic Calculi in AI

- Variants of logic calculi are part of many AI systems
- Logic and logical inference is the base of most types of knowledge representation formalisms (e.g. description logics)
- Most knowledge-based systems (e.g. expert systems) are relying on some type of deductive inference mechanism
- Often, classical logic is not adequate: non-monotonic, probabilistic or fuzzy approaches
(see “Semantische Informationsverarbeitung”)
- Extensions of classical logic for dealing with time or believe: Modal Logic (e.g., BDI-Logic for Multi-agent Systems)

Deductive Planning

- Deductive inference can be used to solve planning problems.
- Introduce a **situation variable** to store the partial plans:

$$s_{i+1} = \text{put}(A, B, s_i), \dots s_2 = \text{puttable}(A, s_1)$$

$$s = \text{put}(A, B, \text{puttable}(A, [\text{on}(A, C), \text{clear}(A) \dots]))$$
- Situation calculus: Introduced by McCarthy (1963) and used for plan construction by resolution by Green (1969)
- In general: extensions of FOL (action languages)
- Proof logically, that a set of goals follows from an initial state given operator definitions (axioms)
- Perform the proof in a **constructive** way (plan is constructed as a byproduct of the proof)

Situation Calculus

A1 $on(a, table, s_1)$

(literal of the initial state)

A2 $\forall S[on(a, table, S) \rightarrow on(a, b, put(a, b, S))]$

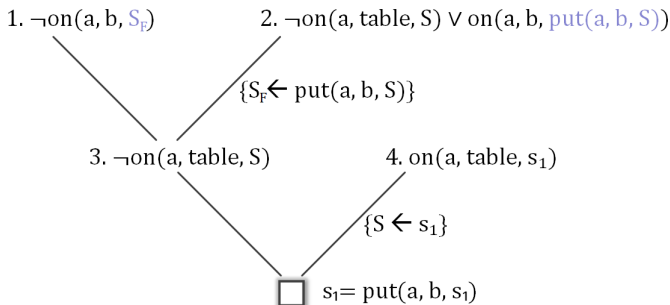
(axiom for put-operator)

\equiv

$\neg on(a, table, S) \vee on(a, b, put(a, b, S))$

(clausal form)

Proof the goal predicate $on(a, b, S_F)$



$s_2 = on(a, table, s_1)$ with $on(a, b, s_2)$ exists and s_2 can be reached by putting a on b in situation

Frame Problem

- No closed world assumption \leftrightarrow full expressive power of FOL
- Problem:
additionally to axioms describing the effects of actions, **frame axioms** become necessary
- Frame axioms are necessary to allow proofing conjunctions of goal literals.
- Example for a frame axiom:
 $\forall S[\text{on}(Y, Z, S) \rightarrow \text{on}(Y, Z, \text{put}(X, Y, S))]$
 $\text{on}(Y, Z, \text{put}(X, Y, S)) \leftarrow \text{on}(Y, Z, S)$
After a block X was put on a block Y , it still holds that Y is lying on a block Z , if this did hold before the action was performed.

Blocksworld in Prolog

Effect Axioms:

$\text{on}(X, Y, \text{put}(X, Y, S)) \leftarrow$	$\text{clear}(X, S) \wedge \text{clear}(Y, S)$
$\text{clear}(Z, \text{put}(X, Y, S)) \leftarrow$	$\text{on}(X, Z, S) \wedge \text{clear}(X, S) \wedge \text{clear}(Y, S)$
$\text{clear}(Y, \text{puttable}(X, S)) \leftarrow$	$\text{on}(X, Y, S) \wedge \text{clear}(X, S)$
$\text{ontable}(X, \text{puttable}(X, S)) \leftarrow$	$\text{clear}(X, S)$

Frame Axioms:

$\text{clear}(X, \text{put}(X, Y, S)) \leftarrow$	$\text{clear}(X, S) \wedge \text{clear}(Y, S)$
$\text{clear}(Z, \text{put}(X, Y, S)) \leftarrow$	$\text{clear}(X, S) \wedge \text{clear}(Y, S) \wedge \text{clear}(Z, S)$
$\text{ontable}(Y, \text{put}(X, Y, S)) \leftarrow$	$\text{clear}(X, S) \wedge \text{clear}(Y, S) \wedge \text{ontable}(Y, S)$
$\text{ontable}(Z, \text{put}(X, Y, S)) \leftarrow$	$\text{clear}(X, S) \wedge \text{clear}(Y, S) \wedge \text{ontable}(Z, S)$
$\text{on}(Y, Z, \text{put}(X, Y, S)) \leftarrow$	$\text{clear}(X, S) \wedge \text{clear}(Y, S) \wedge \text{on}(Y, Z, S)$
$\text{on}(W, Z, \text{put}(X, Y, S)) \leftarrow$	$\text{clear}(X, S) \wedge \text{clear}(Y, S) \wedge \text{on}(W, Z, S)$

Blocksworld in Prolog cont.

Frame Axioms cont.:

$\text{clear}(Z, \text{puttable}(X, S)) \leftarrow$	$\text{clear}(X, S) \wedge \text{clear}(Z, S)$
$\text{ontable}(Z, \text{puttable}(X, S)) \leftarrow$	$\text{clear}(X, S) \wedge \text{ontable}(Z, S)$
$\text{on}(Y, Z, \text{puttable}(X, S)) \leftarrow$	$\text{clear}(X, S) \wedge \text{on}(Y, Z, S)$
$\text{clear}(Z, \text{puttable}(X, S)) \leftarrow$	$\text{on}(Y, X, S) \wedge \text{clear}(Y, S) \wedge \text{clear}(Z, S)$
$\text{ontable}(Z, \text{puttable}(X, S)) \leftarrow$	$\text{on}(Y, X, S) \wedge \text{clear}(Y, S) \wedge \text{ontable}(Z, S)$
$\text{on}(W, Z, \text{puttable}(X, S)) \leftarrow$	$\text{on}(Y, X, S) \wedge \text{clear}(Y, S) \wedge \text{on}(W, Z, S)$

Facts (Initial State):

$\text{on}(d, c, s_1)$	$\text{on}(c, a, s_1)$
$\text{clear}(d, s_1)$	$\text{clear}(b, s_1)$
$\text{ontable}(a, s_1)$	$\text{ontable}(b, s_1)$

Theorem (Goal):

$\text{on}(a, b, S) \wedge \text{on}(b, c, S)$

Planning as Satisfiability Problem (SAT-Planing)

- Propositional satisfiability: given a boolean formula, does there exist an assignment of truth values that makes the formula true (e.g., a model)?
- Very first problem shown to be NP-complete
- Many algorithms exist which work on average case polynomial time (e.g., Davis-Putnam, GSAT)
- Encode a planning problem P with a fixed length solution path n as satisfiability problem Ψ (see lecture Formal Characteristics)
- That is based on set-theoretical representation
- Frame axioms are needed to describe what does not change
- Introduction of additional argument to represent plan-length

Example

- Planning domain:
 - ◇ one robot $r1$
 - ◇ two adjacent locations $l1, l2$
 - ◇ one planning operator
(to move the robot from one location to another)

- Encode (P,n) where $n = 1$

1. Initial state: $\{at(r1, l1)\}$
 Encoding: $at(r1, l1, 0) \wedge \neg at(r1, l2, 0)$

2. Goal: $\{at(r1, l2)\}$
 Encoding: $at(r1, l2, 1) \wedge \neg at(r1, l1, 1)$

3. Operator: see next slide

Example (continued)

- Operator: $\text{move}(r,l,l')$
 precondition: $\text{at}(r,l)$
 effects: $\text{at}(r,l'), \neg\text{at}(r,l)$

Encoding:

$$\begin{array}{l}
 \text{move}(r1, l1, l2, 0) \Rightarrow \text{at}(r1, l1, 0) \wedge \text{at}(r1, l2, 1) \wedge \neg\text{at}(r1, l1, 1) \\
 \text{move}(r1, l2, l1, 0) \Rightarrow \text{at}(r1, l2, 0) \wedge \text{at}(r1, l1, 1) \wedge \neg\text{at}(r1, l2, 1) \\
 \text{move}(r1, l1, l1, 0) \Rightarrow \text{at}(r1, l1, 0) \wedge \text{at}(r1, l1, 1) \wedge \neg\text{at}(r1, l1, 1) \\
 \text{move}(r1, l2, l2, 0) \Rightarrow \text{at}(r1, l2, 0) \wedge \text{at}(r1, l2, 1) \wedge \neg\text{at}(r1, l2, 1) \\
 \left. \begin{array}{l}
 \text{move}(l1, r1, l2, 0) \Rightarrow \dots \\
 \text{move}(l2, r1, l1, 0) \Rightarrow \dots \\
 \text{move}(l1, r2, l1, 0) \Rightarrow \dots \\
 \text{move}(l2, r1, l1, 0) \Rightarrow \dots
 \end{array} \right\} \begin{array}{l}
 \text{contradictions} \\
 \text{(easy to detect)} \\
 \\
 \text{nonsensical, and we can avoid} \\
 \text{generating them if we use data types} \\
 \text{like we did for state-variable} \\
 \text{representation}
 \end{array}
 \end{array}$$

- Operator: $\text{move}(r:\text{robot}, l:\text{location}, l':\text{location})$
 precondition: $\text{at}(r,l)$
 effects: $\text{at}(r,l'), \neg\text{at}(r,l)$

Example (continued)

4. Complete-exclusion axiom:

$$\neg \text{move}(r1, l1, l2, 0) \wedge \neg \text{move}(r1, l2, l1, 0)$$

5. Explanatory frame axioms:

$$\neg \text{at}(r1, l1, 0) \wedge \text{at}(r1, l1, 1) \Rightarrow \text{move}(r1, l2, l1, 0)$$

$$\neg \text{at}(r1, l2, 0) \wedge \text{at}(r1, l2, 1) \Rightarrow \text{move}(r1, l1, l2, 0)$$

$$\text{at}(r1, l1, 0) \wedge \neg \text{at}(r1, l1, 1) \Rightarrow \text{move}(r1, l1, l2, 0)$$

$$\text{at}(r1, l2, 0) \wedge \neg \text{at}(r1, l2, 1) \Rightarrow \text{move}(r1, l2, l1, 0)$$

- Φ is the conjunct of all these

Summary of the Example

- P is a planning problem with one robot and two locations

- ◇ initial state $\{at(r1,l1)\}$
- ◇ goal $\{at(r1,l2)\}$

- Encoding of $(P,1)$

- ◇ $\Phi =$

$[at(r1,l1,0) \wedge \neg at(r1,l2,0)]$	(initial state)
$\wedge [at(r1,l2,0) \wedge \neg at(r1,l1,1)]$	(goal)
$\wedge [move(r1,l1,l2,0) \Rightarrow at(r1,l1,0) \wedge at(r1,l2,1) \wedge \neg at(r1,l1,1)]$	(action)
$\wedge [move(r1,l2,l1,0) \Rightarrow at(r1,l2,0) \wedge at(r1,l1,1) \wedge \neg at(r1,l2,1)]$	(action)
$\wedge [\neg move(r1,l1,l2,0) \vee \neg move(r1,l2,l1,0)]$	(complete exclusion)
$\wedge [\neg at(r1,l1,0) \wedge at(r1,l1,1) \Rightarrow move(r1,l2,l1,0)]$	(frame axiom)
$\wedge [\neg at(r1,l2,0) \wedge at(r1,l2,1) \Rightarrow move(r1,l1,l2,0)]$	(frame axiom)
$\wedge [at(r1,l1,0) \wedge \neg at(r1,l1,1) \Rightarrow move(r1,l1,l2,0)]$	(frame axiom)
$\wedge [at(r1,l2,0) \wedge \neg at(r1,l2,1) \Rightarrow move(r1,l2,l1,0)]$	(frame axiom)

Extracting a Plan

- Let Φ be an encoding of (P, n)
- Suppose we find an assignment of truth values that satisfies Φ .
 - ◇ This means P has a solution of length n
- For $i = 1, \dots, n$, there will be exactly one action a such that $a_i = true$
 - ◇ This is the i 'th action of the plan
- Example:
- The formula on the previous slide
 - ◇ Φ can be satisfied with **move(r1,l1,l2,0)** = *true*
 - ⇒ Thus $\langle move(r1,l1,l2,0) \rangle$ is a solution for $(P, 1)$
 - ◇ It's the only solution - no other way to satisfy Φ

Summary

- Resolution is defined for clausal form
- Logical formula can be rewritten in conjunctive normal form from which a set of clauses can be generated
- Rewriting into clausal form relies on equivalence rules, Skolemization is not an equivalence transformation but a formula and its Skolemization are equivalent w.r.t. satisfiability
- In FOL, identity of formulas can be established by restricting their scope: The most general unifier is defined as the minimal set of substitution of variables by terms to make two formulas equal.
- Resolution is a proof by contradiction.
- For implementing resolution, a strategy to select clauses for refutation is necessary.
- Prolog is a resolution prover based on SLD-resolution.
- Deductive planning is based on search for a constructive proof that the goals are entailed by the effect and frame axioms for a problem domain.
- SAT-planning used efficient SAT-solving algorithms. Problems have to be rewritten in set-theoretic representation and frame axioms have to be included.