Intelligent Agents
Representations for Classical Planning

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Quick Review: The DWR Domain
Motivation

Representation of a Planning Problem

To compute a plan you need to have a representation of the environment, an initial state and a set of goals to be reached (objectives)

Diagram:
- Planning problem
- Description of \( \Sigma \)
- Initial state
- Objectives
- Execution status
- Plans
- Controller
- Observations
- Actions
- System \( \Sigma \)
- Events
Outline

- Representation schemes
  - Classical representation
  - Set-theoretic representation
  - State-variable representation
  - Examples: DWR and the Blocks World
  - Comparison of representation formats
Classical Representation

- Start with a *function-free* first-order language
  - Finitely many predicate symbols and constant symbols, but no function symbols

- Example: the DWR domain
  - Locations: l1, l2, ...
  - Containers: c1, c2, ...
  - Piles: p1, p2, ...
  - Robot carts: r1, r2, ...
  - Cranes: k1, k2, ...

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Classical Representation

- **Atom**: predicate symbol and args
  
  Use these to represent both fixed and dynamic relations
  
  - adjacent($l,l'$)
  - occupied($l$)
  - loaded($r,c$)
  - holding($k,c$)
  - in($c,p$)
  - top($c,p$)
  - attached($p,l$)
  - at($r,l$)
  - unloaded($r$)
  - empty($k$)
  - on($c,c'$)
  - top($pallet,p$)
  
- **Ground** expression: contains no variable symbols - e.g., in($c1,p3$)
- **Unground** expression: at least one variable symbol - e.g., in($c1,x$)
- **Substitution**: $\theta = \{x_1 \leftarrow v_1, x_2 \leftarrow, ..., x_n \leftarrow v_n\}$
  
  - Each $x_i$ is a variable symbol; each $v_i$ is a term

- **Instance of $e$**: result of applying a substitution $\theta$ to $e$
  
  - Replace variables of $e$ simultaneously, not sequentially
**States**

- **State**: a set $s$ of ground atoms
  - The atoms represent the things that are true in one of $\Sigma$’s states
  - Only finitely many ground atoms, so only finitely many possible states

$$s_1 = \{\text{attached}(p1, loc1), \text{in}(c1, p1), \text{in}(c3, p1), \text{top}(c3, p1), \text{on}(c3, c1), \text{on}(c1, pallet), \text{attached}(p2, loc1), \text{in}(c2, p2), \text{top}(c2, p2), \text{on}(c2, pallet), \text{belong}(\text{crane1}, \text{loc1}), \text{empty}(\text{crane1}), \text{adjacent}(\text{loc1}, \text{loc2}), \text{adjacent}(\text{loc2}, \text{loc1}), \text{at}(r1, \text{loc2}), \text{occupied}(\text{loc2}), \text{unloaded}(r1)\}$$
Operators

- **Operator**: a triple \( o=(\text{name}(o), \text{precond}(o), \text{effects}(o)) \)
  - **name** is a syntactic expression of the form \( n(x_1, \ldots, x_k) \)
    - \( n: \) operator symbol - must be unique for each operator
    - \( x_1, \ldots, x_k: \) variable symbols (parameters)
      - must include every variable symbol in \( o \)
  - **precond**: 
    - literals that must be true in order to use the operator
  - **effects**: 
    - literals the operator will make true

\[
\text{take}(k, l, c, d, p) \]

;; crane \( k \) at location \( l \) takes \( c \) off of \( d \) in pile \( p \)

**precond**: belong\((k, l)\), attached\((p, l)\), empty\((k)\), top\((c, p)\), on\((c, d)\)

**effects**: holding\((k, c)\), \( \neg \text{empty}(k)\), \( \neg \text{in}(c, p)\), \( \neg \text{top}(c, p)\), \( \neg \text{on}(c, d)\), top\((d, p)\)
**Actions**

- **Action**: ground instance (via substitution) of an operator
  
  **Operator** (i.e., an abstract schema of actions, containing variable)

  \[
  \text{take}(k, l, c, d, p)
  \]
  ;; crane \( k \) at location \( l \) takes \( c \) off of \( d \) in pile \( p \)
  precond: \( \text{belong}(k, l), \text{attached}(p, l), \text{empty}(k), \text{top}(c, p), \text{on}(c, d) \)
  effects: \( \text{holding}(k, c), \neg \text{empty}(k), \neg \text{in}(c, p), \neg \text{top}(c, p), \neg \text{on}(c, d), \text{top}(d, p) \)

- **Action** (i.e., an instantiated operator which can be applied to some specific states)

  \[
  \text{take}(\text{crane1}, \text{loc1}, c3, c1, p1)
  \]
  ;; crane \( \text{crane1} \) at location \( \text{loc1} \) takes \( c3 \) off of \( c1 \) in pile \( p1 \)
  precond: \( \text{belong}(<\text{crane1}, \text{loc1}>, \text{attached}(p1, \text{loc1}), \text{empty}(\text{crane1}), \text{top}(c3, p1), \text{on}(c3, c1) \)
  effects: \( \text{holding}(\text{crane1}, c3), \neg \text{empty}(\text{crane1}), \neg \text{in}(c3, p1), \neg \text{top}(c3, p1), \neg \text{on}(c3, c1), \text{top}(c1, p1) \)
Let $S$ be a set of literals. Then
- $S^+ = \{ \text{atoms that appear positively in } S \}$
- $S^- = \{ \text{atoms that appear negatively in } S \}$

More specifically, let $a$ be an operator or action. Then
- $\text{precond}^+(a) = \{ \text{atoms that appear positively in } a's \text{ preconditions} \}$
- $\text{precond}^-(a) = \{ \text{atoms that appear negatively in } a's \text{ preconditions} \}$
- $\text{effects}^+(a) = \{ \text{atoms that appear positively in } a's \text{ effects} \}$
- $\text{effects}^-(a) = \{ \text{atoms that appear negatively in } a's \text{ effects} \}$

$\text{take}(k, l, c, d, p)$
- ;; crane $k$ at location $l$ takes $c$ off of $d$ in pile $p$
- precond: $\text{belong}(k, l), \text{attached}(p, l), \text{empty}(k), \text{top}(c, p), \text{on}(c, d)$
- effects: $\text{holding}(k, c), \neg \text{empty}(k), \neg \text{in}(c, p), \neg \text{top}(c, p), \neg \text{on}(c, d), \text{top}(d, p)$

Let $S$ be a set of literals. Then
- $\text{effects}^+(\text{take}(k, l, c, d, p)) = \{ \text{holding}(k, c), \text{top}(d, p) \}$
- $\text{effects}^-(\text{take}(k, l, c, d, p)) = \{ \text{empty}(k), \text{in}(c, p), \text{top}(c, p), \text{on}(c, d) \}$
An action \(a\) is \textit{applicable} to a state \(s\) if \(s\) satisfies \(\text{precond}(a)\), i.e., if \(\text{precond}^+(a) \subseteq s\) and \(\text{precond}^-(a) \cap s = \emptyset\).

Here are an action and a state that it’s applicable to:

\[
s_1 = \{\text{attached}(p_1, \text{loc}1), \text{in}(c_1, p_1), \text{in}(c_3, p_1), \text{top}(c_3, p_1), \text{on}(c_3, c_1), \text{on}(c_1, \text{pallet}), \text{attached}(p_2, \text{loc}1), \text{in}(c_2, p_2), \text{top}(c_2, p_2), \text{on}(c_2, \text{pallet}), \text{belong}(\text{crane}1, \text{loc}1), \text{empty}(\text{crane}1), \text{adjacent}(\text{loc}1, \text{loc}2), \text{adjacent}(\text{loc}2, \text{loc}1), \text{at}(r_1, \text{loc}2), \text{occupied}(\text{loc}2), \text{unloaded}(r_1)\}
\]

\(\text{take(\text{crane}1, \text{loc}1, c_3, c_1, p_1)}\)

\text{;; crane crane1 at location loc1 takes c3 off of c1 in pile p1}

\(\text{precond: belong(\text{crane}1,\text{loc}1), attached(p1,\text{loc}1), empty(\text{crane}1), top(c3,p1), on(c3,c1)}\)

\(\text{effects: holding(\text{crane}1, c3), \neg empty(\text{crane}1), \neg in(c3,p1), \neg top(c3,p1), \neg on(c3,c1), top(c1,p1)}\)
If $a$ is applicable to $s$, the result of performing it is

$$\gamma(s, a) = s - \text{effects}^- (a) \cup \text{effects}^+ (a)$$

Delete the negative effects, and add the positive ones

```
take(crane1, loc1, c3, c1, p1)
;; crane crane1 at location loc1 takes c3 off of c1 in pile p1
precond: belong(crane1,loc1), attached(p1,loc1),
        empty(crane1), top(c3,p1), on(c3,c1)
effects: holding(crane1, c3), \neg empty(crane1), \neg in(c3,p1),
        \neg top(c3,p1), \neg on(c3,c1), top(c1,p1)
```
\[ s_1 = \{ attached(p_1, loc_1), in(c_1, p_1), in(c_3, p_1), top(c_3, p_1), on(c_3, c_1), \\
\quad on(c_1, \text{pallet}), attached(p_2, loc_1), in(c_2, p_2), top(c_2, p_2), on(c_2, \text{pallet}), \\
\quad belong(crane_1, loc_1), empty(crane_1), adjacent(loc_1, loc_2), \\
\quad adjacent(loc_2, loc_1), at(r_1, loc_2), occupied(loc_2), unloaded(r_1) \} \]

take(crane_1, loc_1, c_3, c_1, p_1) with 
precond: belong(crane_1, loc_1), attached(p_1, loc_1), empty(crane_1), top(c_3, p_1), on(c_3, c_1)
is applicable to \( s_1 \):
\[
\{ belong(crane_1, loc_1), attached(p_1, loc_1), empty(crane_1), top(c_3, p_1), on(c_3, c_1) \subseteq s_1 \} = \\
\{ attached(p_1, loc_1), in(c_1, p_1), in(c_3, p_1), top(c_3, p_1), on(c_3, c_1), \\
\quad on(c_1, \text{pallet}), attached(p_2, loc_1), in(c_2, p_2), top(c_2, p_2), on(c_2, \text{pallet}), \\
\quad belong(crane_1, loc_1), empty(crane_1), adjacent(loc_1, loc_2), \\
\quad adjacent(loc_2, loc_1), at(r_1, loc_2), occupied(loc_2), unloaded(r_1) \}
\]

Application:
\[ s_1 = \{ attached(p_1, loc_1), in(c_1, p_1), in(c_3, p_1), top(c_3, p_1), on(c_3, c_1), \\
\quad on(c_1, \text{pallet}), attached(p_2, loc_1), in(c_2, p_2), top(c_2, p_2), on(c_2, \text{pallet}), \\
\quad belong(crane_1, loc_1), empty(crane_1), adjacent(loc_1, loc_2), \\
\quad adjacent(loc_2, loc_1), at(r_1, loc_2), occupied(loc_2), unloaded(r_1) \\
\quad \cup holding(crane_1, c_3), top(c_1, p_1) \} \]
move(r, l, m)
    ;; robot r moves from location l to location m
    precond: adjacent(l,m), at(r,l), ¬occupied(m)
    effects:  at(r,m), occupied(m), ¬occupied(l), ¬at(r,l)

load(k, l, c, r)
    ;; crane k at location l loads container c onto robot r
    precond: belong(k,l), holding(k,c), at(r,l), unloaded(r)
    effects:  empty(k), ¬holding(k,c), loaded(r,c), ¬unloaded(r)

unload(k, l, c, r)
    ;; crane k at location l takes container c from robot r
    precond: belong(k,l), at(r,l), loaded(r,c), empty(k)
    effects:  ¬empty(k), holding(k,c), unloaded(r), ¬loaded(r)

put(k, l, c, d, p)
    ;; crane k at location l puts c onto d in pile p
    precond: belong(k,l), attached(p,l), holding(k,c), top(d,p)
    effects:  ¬holding(k,c),empty(k), in(c,p), top(c,p),on(c,d),¬top(d,p)

take(k, l, c, d, p)
    ;; crane k at location l takes c off of d in pile p
    precond: belong(k,l), attached(p,l), empty(k), top(c,p), on(c,d)
    effects:  holding(k,c),¬empty(k), ¬in(c,p), ¬top(c,p),¬on(c,d),¬top(d,p)

Planning domain:
language plus operators

- Corresponds to a set of state-transition systems
- Example: operators for the DWR domain
Planning Problems

- Given a planning domain (language $L$, operators $O$)
  - *Statement* of a planning problem: a triple $P = (O, s_0, g)$
    - $O$ is the collection of operators
    - $s_0$ is a state (the initial state)
    - $g$ is a set of literals (the goal formula)
  - The actual *planning problem*: $P = (\Sigma, s_0, S_g)$
    - $s_0$ and $S_g$ are as above
    - $\Sigma = (S, A, \gamma)$ is a state-transition system
    - $S = \{\text{all sets of ground atoms in } L\}$
    - $A = \{\text{all ground instances of operators in } O\}$
    - $\gamma = \text{the state-transition function determined by the operators}$

- I’ll often say “planning problem“ when I mean the statement of the problem
Plans and Solutions

- **Plan**: any sequence of actions $\sigma = \langle a_1, a_2, \ldots, a_n \rangle$ such that each $a_i$ is a ground instance of an operator in $O$

- The plan is a **solution** for $P = (O, s_0, g)$ if it is executable and achieves $g$
  - i.e., if there are states $s_0, s_1, \ldots, s_n$ such that
    
    $\Rightarrow \gamma(s_0, a_1) = s_1$
    $\Rightarrow \gamma(s_1, a_2) = s_2$
    $\Rightarrow \ldots$
    $\Rightarrow \gamma(s_{n-1}, a_n) = s_n$
    $\Rightarrow s_n$ satisfies $g$
Example

Let $P_1 = (O, s_1, g_1)$, where

- $O$ is the set of operators given earlier

- $s_1 = \{ \text{attached}(p1, loc1), \text{in}(c1,p1), \text{in}(c3,p1), \text{top}(c3,p1), \text{on}(c3,c1), \text{on}(c1,pallet), \text{attached}(p2,loc1), \text{in}(c2,p2), \text{top}(c2,p2), \text{on}(c2,pallet), \text{belong}(\text{crane1},\text{loc1}), \text{empty}(\text{crane1}), \text{adjacent}(\text{loc1},\text{loc2}), \text{adjacent}(\text{loc2},\text{loc1}), \text{at}(r1,\text{loc2}), \text{occupied}(\text{loc2}), \text{unloaded}(r1) \}$

- $g_1 = \{ \text{loaded}(r1, c3), \text{at}(r1, \text{loc2}) \}$
Here are three solutions for $P_1$:

1. $\langle \text{take}(\text{crane1}, \text{loc1}, \text{c3}, \text{c1}, \text{p1}), \text{move}(\text{r1}, \text{loc2}, \text{loc1}), \text{move}(\text{r1}, \text{loc1}, \text{loc2}), \text{move}(\text{r1}, \text{loc2}, \text{loc1}), \text{load}(\text{crane1}, \text{loc1}, \text{c3}, \text{r1}), \text{move}(\text{r1}, \text{loc1}, \text{loc2}) \rangle$

2. $\langle \text{take}(\text{crane1}, \text{loc1}, \text{c3}, \text{c1}, \text{p1}), \text{move}(\text{r1}, \text{loc2}, \text{loc1}), \text{load}(\text{crane1}, \text{loc1}, \text{c3}, \text{r1}), \text{move}(\text{r1}, \text{loc1}, \text{loc2}) \rangle$

3. $\langle \text{move}(\text{r1}, \text{loc2}, \text{loc1}), \text{take}(\text{crane1}, \text{loc1}, \text{c3}, \text{c1}, \text{p1}), \text{load}(\text{crane1}, \text{loc1}, \text{c3}, \text{r1}), \text{move}(\text{r1}, \text{loc1}, \text{loc2}) \rangle$

Each of them produces the state shown here:
The first is *redundant*: can remove actions and still have a solution

- \( \langle \text{take(crane1,loc1,c3,c1,p1), move(r1,loc2,loc1), move(r1,loc1,loc2),move(r1,loc2,loc1), load(crane1,loc1,c3,r1), move(r1,loc1,loc2)} \rangle \)
- \( \langle \text{take(crane1,loc1,c3,c1,p1), move(r1,loc2,loc1), load(crane1,loc1,c3,r1), move(r1,loc1,loc2)} \rangle \)
- \( \langle \text{move(r1,loc2,loc1), take(crane1,loc1,c3,c1,p1), load(crane1,loc1,c3,r1), move(r1,loc1,loc2)} \rangle \)

The 2nd and 3rd are *irredundant* and *shortest*
Set-Theoretic Representation

- Like classical representation, but restricted to propositional logic

- States:
  - Instead of a collection of ground atoms ... 
    \{on(c1,pallet), on(c1,r1), on(c1,c2), \ldots, at(r1,l1), at(r1,l2), \ldots \}
  - ... use a collection of propositions (boolean variables):
    \{on-c1-pallet, on-c1-r1, on-c1-c2, \ldots, at-r1-l1, at-r1-l2, \ldots \}

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Instead of operators like this one,

\[ \text{take}(k, l, c, d, p) \]

;; crane \( k \) at location \( l \) takes \( c \) off of \( d \) in pile \( p \)

precond: \( \text{belong}(k, l) \), \( \text{attached}(p, l) \), \( \text{empty}(k) \), \( \text{top}(c, p) \), \( \text{on}(c, d) \)

effects: \( \text{holding}(k, c) \), \( \neg \text{empty}(k) \), \( \neg \text{in}(c, p) \), \( \neg \text{top}(c, p) \), \( \neg \text{on}(c, d) \), \( \neg \text{top}(d, p) \)

\[ \text{take}(\text{crane1}, \text{loc1}, c3, c1, p1) \]

;; crane \( \text{crane1} \) at location \( \text{loc1} \) takes \( c3 \) off of \( c1 \) in pile \( p1 \)

precond: \( \text{belong}(\text{crane1}, \text{loc1}) \), \( \text{attached}(p1, \text{loc1}) \), \( \text{empty}(\text{crane1}) \), \( \text{top}(c3, p1) \), \( \text{on}(c3, c1) \)

effects: \( \text{holding}(\text{crane1}, c3) \), \( \neg \text{empty}(\text{crane1}) \), \( \neg \text{in}(c3, p1) \), \( \neg \text{top}(c3, p1) \), \( \neg \text{on}(c3, c1) \), \( \text{top}(c1, p1) \)

\[ \text{take} - \text{crane1} - \text{loc1} - c3 - c1 - p1 \]

precond: \( \text{belong-crate1-loc1} \), \( \text{attached-p1-loc1} \), \( \text{empty-crane1} \), \( \text{top-c3-p1} \), \( \text{on-c3-c1} \)

delete: \( \text{empty-crane1} \), \( \text{in-c3-p1} \), \( \text{top-c3-p1} \), \( \text{on-c3-p1} \)

add: \( \text{holding-crane1-c3} \), \( \text{top-c1-p1} \)
Remarks on Set-Theoretic Representation

A set-theoretic representation is equivalent to a classical representation in which all of the atoms are ground.

Exponential blow-up
- If a classical operator contains $n$ atoms and each atom has arity $k$, then it corresponds to $c^{nk}$ actions where $c = |\{\text{constant symbols}\}|$.

Set-theoretic representation is applied in practice for SAT-solvers and model-checking. For these applications typically, a set-theoretical representation is algorithmically generated from a classical representation.

Set-theoretic representation is a very useful concept for theoretical analyses of planning (using an explicit representation of all states of a problem).
State-Variable Representation

- Use ground atoms for properties that do not change, e.g., adjacent(loc1,loc2)
- For properties that can change, assign values to state variables
  - Like fields in a record structure
- Classical and state-variable representations take similar amounts of space
  - Each can be translated into the other in low-order polynomial time

move(r,l,m)

;; robot r at location l moves to an adjacent location m
precond: rloc(r)=l, adjacent(l,m)
effects: rloc(r) ← m

{top(p1)=c3, cpos(c3)=c1, cpos(c1)=pallet, holding(cran1)=nil, rloc(r1)=loc2, loaded(r1)=nil, ...}
Remarks on State-Variable Representation

- The engineering view rather than the logic view
- Especially useful when a new value which is assigned to a variable has to be calculated by some procedure
- Example: Depots Domain with Fluents in PDDL
  - Predefined functions assign and increase
- In general, arbitrary user-defined methods can be used:
  
  ```
  fuel(r) ← calcDecrease(r,l,m)
  ```
(define (domain Depot-object-fluents)
  (:requirements :typing :equality :fluents)
  (:types place locatable - object
depot distributor - place
truck hoist surface - locatable
pallet crate - surface)

  (:constants no-crate - crate)
  (:predicates (clear ?s - surface))
  (:functions
   (load-limit ?t - truck)
   (current-load ?t - truck)
   (weight ?c - crate)
   (fuel-cost) - number
   (position-of ?l - locatable) - place
   (crate-held ?h - hoist) - crate
   (thing-below ?c - crate) - (either surface truck))

  (:action drive
   :parameters (?t - truck ?p - place)
   :effect (and (assign (position-of ?t) ?p)
                (increase (fuel-cost) 10))))
Example: The Blocks World

- Infinitely wide table, finite number of children’s blocks
- Ignore where a block is located on the table
- A block can sit on the table or on another block
- Want to move blocks from one configuration to another
  - e.g.,

![Diagram of initial state and goal with blocks a, b, c, d, and e]

- Can be expressed as a special case of DWR
  - But the usual formulation is simpler
- I’ll give classical, set-theoretic, and state-variable formulations
  - For the case where there are five blocks
### Classical Representation: Symbols

- **Constant symbols:**
  - The blocks: \( a, b, c, d, e \)

- **Predicates:**
  - \( ontable(x) \) - block \( x \) is on the table
  - \( on(x,y) \) - block \( x \) is on block \( y \)
  - \( clear(x) \) - block \( x \) has nothing on it
  - \( holding(x) \) - the robot hand is holding block \( x \)
  - \( handempty \) - the robot hand isn’t holding anything
Classical Operators

\textbf{unstack}(x,y)
Precond: \(\text{on}(x,y), \text{clear}(x), \text{handempty}\)
Effects: \(\neg\text{on}(x,y), \neg\text{clear}(x), \neg\text{handempty}, \text{holding}(x), \text{clear}(y)\)

\textbf{stack}(x,y)
Precond: \(\text{holding}(x), \text{clear}(y)\)
Effects: \(\neg\text{holding}(x), \neg\text{clear}(y), \\text{on}(x,y), \text{clear}(x), \text{handempty}\)

\textbf{pickup}(x)
Precond: \(\text{ontable}(x), \text{clear}(x), \text{handempty}\)
Effects: \(\neg\text{ontable}(x), \neg\text{clear}(x), \neg\text{handempty}, \text{holding}(x)\)

\textbf{putdown}(x)
Precond: \(\text{holding}(x)\)
Effects: \(\neg\text{holding}(x), \text{ontable}(x), \text{clear}(x), \text{handempty}\)
Set-Theoretic Representation: Symbols

- For five blocks, there are 36 propositions
- Here are 5 of them:
  - `ontable-a` - block $a$ is on the table
  - `on-c-a` - block $c$ is on block $a$
  - `clear-c` - block $c$ has nothing on it
  - `holding-d` - the robot hand is holding block $d$
  - `handempty` - the robot hand isn’t holding anything
Set-Theoretic Actions

Fifty different actions

Here are four of them:

**unstack-c-a**
- Pre: on-c,a, clear-c, handempty
- Del: on-c,a, clear-c, handempty
- Add: holding-c, clear-a

**stack-c-a**
- Pre: holding-c, clear-a
- Del: holding-c, clear-a
- Add: on-c-a, clear-c, handempty

**pickup-c**
- Pre: ontable-c, clear-c, handempty
- Del: ontable-c, clear-c, handempty
- Add: holding-c

**putdown-c**
- Pre: holding-c
- Del: holding-c
- Add: ontable-c, clear-c, handempty
State-Variable Representation: Symbols

- **Constant symbols:**
  - $a, b, c, d, e$ of type *block*
  - $0, 1, \text{table, nil}$ of type *other*

- **State variables:**
  - $\text{pos}(x)=y$ if block $x$ is on the block $y$
  - $\text{pos}(x)=\text{table}$ if block $x$ is on the table
  - $\text{pos}(x)=\text{nil}$ if block $x$ is being held
  - $\text{clear}(x)=1$ if block $x$ has nothing on it
  - $\text{clear}(x)=0$ if block $x$ is being held or has another block on it
  - $\text{holding}=x$ if the robot hand is holding block $x$
  - $\text{holding}=\text{nil}$ if the robot hand is holding nothing
State-Variable Operators

**unstack(x : block, y : block)**
- Precond: $\text{pos}(x)=y$, $\text{clear}(y)=0$, $\text{clear}(x)=1$, $\text{holding}=\text{nil}$
- Effects: $\text{pos}(x)=\text{nil}$, $\text{clear}(x)=0$, $\text{holding}=x$, $\text{clear}(y)=1$

**stack(x : block, y : block)**
- Precond: $\text{holding}=x$, $\text{clear}(x)=0$, $\text{clear}(y)=1$
- Effects: $\text{holding}=\text{nil}$, $\text{clear}(y)=0$, $\text{pos}(x)=y$, $\text{clear}(x)=1$

**pickup(x : block)**
- Precond: $\text{pos}(x)=\text{table}$, $\text{clear}(x)=1$, $\text{holding}=\text{nil}$
- Effects: $\text{pos}(x)=\text{nil}$, $\text{clear}(x)=0$, $\text{holding}=x$

**putdown(x : block)**
- Precond: $\text{holding}=x$
- Effects: $\text{holding}=\text{nil}$, $\text{pos}(x)=\text{table}$, $\text{clear}(x)=1$
Expressive Power

- Any problem that can be represented in one representation can also be represented in the other two.
- Can convert in linear time and space, except when converting to set-theoretic (where we get an exponential blow-up).
A function \( f : D^n \Rightarrow W \)
is a unique mapping from \((n)\) values of domains to a value of a range

Examples:

- \( \text{succ} : \text{Nat} \rightarrow \text{Nat} \)
  - \( \text{succ}(0) = 1, \text{succ}(1) = 1, \ldots \)

- \( \text{plus} : \text{Nat} \times \text{Nat} \rightarrow \text{Nat} \)
  - \( \text{plus}(0, 0) = 0, \text{plus}(0, 1) = 1, \ldots \)
Functions and Relations cont.

- A relation $R$ is a relationship between sets
- Examples:
  - $R_{\text{succ}} \subseteq \text{Nat} \times \text{Nat} = \{(0, 1), (0, 2), (0, 3), \ldots\}$
  - $R_{\text{plus}} \subseteq \text{Nat} \times \text{Nat} \times \text{Nat} = \{(0, 0, 0), (0, 1, 1), \ldots\}$
- Each function can be transformed into a relation but not each relation is a function!

Append in the functional language Lisp:

```lisp
(defun append (L1, L2)
  (cond ((null L1) L2)
        (T (cons (car L1) (append (cdr L1) L2))))
)
```

Append in the logical language Prolog:

```prolog
append([], L, L).
append([X|Xs], L2, [X|L]) :- append(Xs, L2, L).
```
Comparison

- Classical representation
  - The most popular for classical planning, partly for historical reasons

- Set-theoretic representation
  - Can take much more space than classical representation
  - Useful in algorithms that manipulate ground atoms directly
    \(\Rightarrow\) e.g., planning graphs, satisfiability
  - Useful for certain kinds of theoretical studies

- State-variable representation
  - Without usage of arbitrary usr-defined functions equivalent to classical representation in expressive power
  - Less natural for logicians, more natural for engineers
  - Useful in non-classical planning problems as a way to handle numbers, functions, time
Extended Representations

- Typed variables and relations
- Conditional Operators
- Quantified Expressions
- Equality Constraints
- Disjunctive Preconditions
- Function Symbols
- Axiomatic Inference
- Attached Procedures

- are built upon the classical representation
- can be represented in PDDL
- are explored in some planning systems
Summary

- In classical representation a state is a set of ground literals
- The problem domain is defined by a set of operators representing the possible actions
- An action is a fully instantiated operator
- An action can be applied to a state if all positive preconditions hold, e.g. are included in the state (and if no negative precondition holds)
- The effect of an action is calculated by deleting literals which no longer hold after the execution of the action and by inserting literals which hold after the execution
- This simple way to model state-transitions can be realized because of the closed-world assumption
Set-theoretic representations can be generated by writing all ground instances (exponential blow-up)

This representation is useful because there exist efficient algorithms for manipulating ground atoms (SAT-solvers, model checking)

The state-variable representation works with assignments of values to state-variables. This is useful when dealing with numbers (time, functions, ...)