AI-KI-B

Problem Solving by Search in State-Spaces

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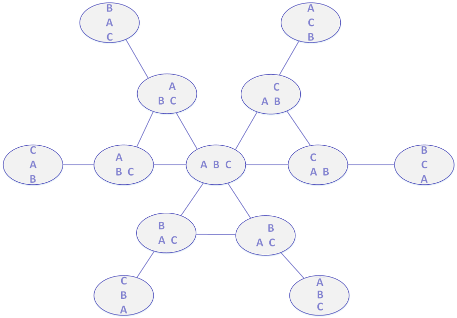
In a nutshell: AI Search Algorithms

- Problem solving, planning, automated inference – and most other areas of AI – depend on intelligent search
- Basic concept: **state space** (or problem space)
  an abstract representation of the (real) states in the world and (real) actions which allow to reach one state from another



e.g.: initial state *tower 'C A B'* and goal state *tower 'A B C'* admissible sequence of actions: put C on table, put B on table, put A on table, put B on C, put A on B

In a nutshell: AI Search Algorithms

- Problem solving: search for a (shortest) path from an initial state to a state which fulfills a set of problem solving goals
- AI search algorithms mostly are variants of depth-first search where only a small part of states is explored – because state spaces are much too large to apply standard graph search algorithms (e.g. the Dijkstra algorithm)
- A good heuristics can reduce search effort dramatically. However, the heuristics must be carefully designed in a way that
    - the (shortest) solution has a chance to be found (**completeness**) and that
    - the result is indeed an admissible solution (**soundness**).

Outline

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  - **Breadth-First Search (BFS)**
- **Complexity of Blocks-World**
- **Cost-based Optimal Search**
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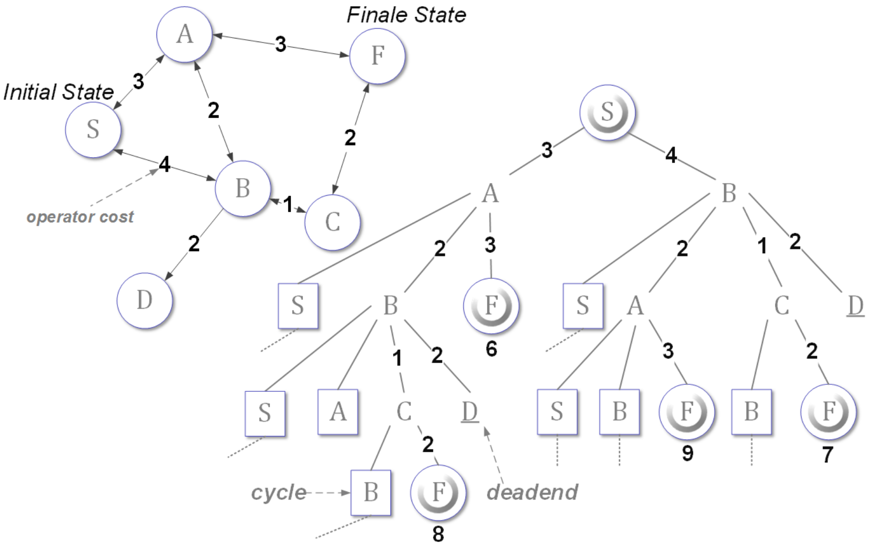
- Cost Estimation (Heuristic Function)
- Heuristic Search Algorithms
  - Hill Climbing (Depth-First, Greedy)
  - Branch and Bound Algorithms (BFS-based)
    - Best First Search
    - A*
- Generic Algortithm for Graph Search
- Designing Heuristic Functions

Running Example

- In the following: We are not concerned with how a single state transformation is calculated (see state-space planning)
- We represent problems as graphs with abstract states (nodes) and abstract actions (arcs), i.e. state spaces
- If we label arcs with numbers, the numbers represent **costs** of the different actions (time, resources).
- Illustration: Navigation problem with nodes as cities and arcs as direct connections; blocksworld problems with nodes as constellations of blocks and arcs as put/puttable operators (might have different costs for different blocks)
- Please note: In general the state-space (graph) is not given explicitly!
- A part of the state space is constructed during search (the states which we explore) in form of a **search tree**.

Example Search Tree



Search Algorithms

- There exist different **search strategies**:
  - **Basic, uninformed ('blind') methods**:
    random search, systematic strategies (depth-first, breadth-first)
  - **Search algorithms for operators with different costs**
  - **Heuristic search**:
    use assumptions to guide the selection of the next candidate operator

Search Tree

- During search for a solution, starting from an initial state, a search tree is generated.
    - **Root**: initial state
    - Each **path from the root to a leaf**: (partial) solution
    - **Intermediate nodes**: intermediate states
    - **Leafs**: Final states or dead ends
- If the same state appears more than once on a path, we have a **cycle**. Without cycle-check search might not terminate! (infinite search tree)

Overall Search Strategy

As long as no final state is reached or there are still reachable, yet unexplored states:

- Collect all operators which can be applied in the current state
  Match state with application conditions
- **Select** on applicable operator.
  In our example: alphabetic order of the labels of the resulting node.
  In general: give a preference order
- **Apply** the operator, generating a successor state

Remark:

$\Rightarrow$ The **Match-Select-Apply Cycle** is the core of “production systems” (see human problem solving)

$\Rightarrow$ Select is realized with respect to the different search algorithms

Problem Solving

- A **problem** is defined by the following components:
  - Initial state
  - Actions or successor function $S(x)$, can be associated with costs
  - Goal test: explicit (current state equals goal state or current state includes all goals) or implicit (as boolean test, e.g. checkmate(x))
- A **problem solution** is a sequence of actions leading from the initial state to a goal state
- Problem solving and planning
  - Problem solving: focus on search algorithms
  - Planning: language for representing problem domains and problems + planning (i.e. search) algorithm (e.g. PDDL as representation language, FF-plan as algorithm)

State-Space and Admissibility

- A search algorithm generates a sequence of actions.
- This sequence of actions is called a solution of a problem or admissible,
    - if the sequence transforms the initial state in a goal state and
    - if all states on the solution path are states which are possible in the problem domain.
- Proofs of formal characteristics of a search algorithm (e.g. soundness and completeness) are based on the notion of state-space as the world (semantic) in which the action sequence is executed.
- The concept of a state space has been introduced by the AI pioneers Newell and Simon under the name of a problem space.

Depth-First Search

- Construct *one* path from initial to final state.
- Backtracking:
  Go back to predecessor state and try to generate another successor state (if none exists, backtrack again etc.), if:
  - the reached state is a dead-end, or
  - the reached state was already reached before (cycle)
- Data structure to store the already explored states:
  Stack; depth-first is based on a “last in first out” (LIFO) strategy
- Cycle check: does the state already occur in the path.
- Result: in general not the shortest path (but the first path)

Effort of Depth-First Search

- In the best-case, depth-first search finds a solution in linear time $O(d)$, for d as average depth of the search tree: Solution can be found on a path of depth d and the first path is already a solution.
- In the worst-case, the complete search-tree must be generated: The problem has only one possible solution and this path is created as the last one during search; or the problem has no solution and the complete state-space must be explored.

Remark: Depth-First Search

- The most parsimonious way to store the (partial) solution path is to push always only the current state on the stack.
  Problem: additional infrastructure for backtracking
  (remember which operators were already applied to a fixed state)
- In the following: push always the partial solution path to the stack.

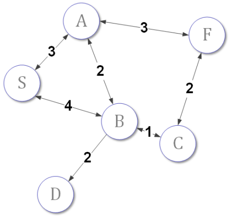
Depth-First Algorithm

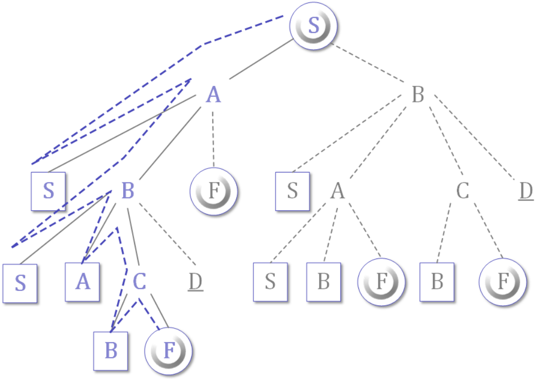
Winston, 1992
To conduct a depth-first search,

- Form a one-element stack consisting of a zero-length path that contains only the root node.
- Until the top-most path in the stack terminates at the goal node or the stack is empty,
    - Pop the first path from the stack; create new paths by extending the first path to all neighbors of the terminal node.
    - Reject all new paths with loops.
    - Push the new paths, if any, on the stack.
- If the goal node is found, announce success; otherwise announce failure.

Depth-First Example

((S))
((S A) (S B))
((S A B) (S A F) [(S A S)] (S B))
((S A B C) (S A B D) [(S A B S)] (S A F) (S B))
([(S A B C B)] (S A B C F) (S A B D) (S A F) (S B))





Breadth-First Search

- The search tree is expanded level-wise.
- No backtracking necessary.
- Data structure to store the already explored states:
  Queue breadth-first is based on a “first in first out” (FIFO) strategy
- Cycle check:
  for finite state-spaces not necessary for termination (but for efficiency)
- Result: shortest solution path
- Effort: If a solution can be first found on level d of the search tree and for an average branching factor b: $O(b^d)$

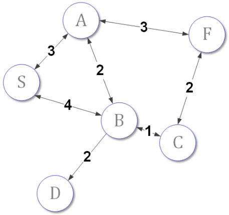
Breadth-First Algorithm

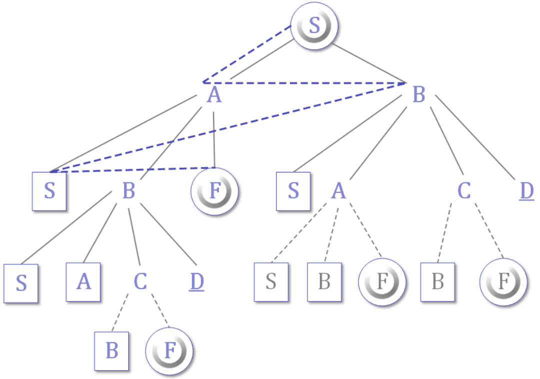
Winston, 1992
To conduct a breadth-first search,

- Form a one-element queue consisting of a zero-length path that contains only the root node.
- Until the first path in the queue terminates at the goal node or the queue is empty,
    - Remove the first path from the queue; create new paths by extending the first path to all neighbors of the terminal node.
    - Reject all new paths with loops.
    - Add the new paths, if any, to the *back* of the queue.
- If the goal node is found, announce success; otherwise announce failure.

Breath-First Example

((S))
((S A) (S B))
((S B) (S A B) (S A F) [(S A S)])
((S A B) (S A F) (S B A) (S B C) (S B D) [(S B S)])
((S A F) (S B A) (S B C) (S B D) (S A B C) (S A B D) [(S A B S)])





Complexity of Blocksworld Problems

Remember: Problems can be characterized by their **complexity**, most problems considered in AI are NP-hard.

# blocks	1	2	3	4	5
# states	1	3	13	73	501
approx.	1.0×10^0	3.0×10^0	1.3×10^1	7.3×10^1	5.0×10^2
# blocks	6	7	8	9	10
# states	4051	37633	394353	4596553	58941091
approx.	4.1×10^3	3.8×10^4	3.9×10^5	4.6×10^6	5.9×10^7
#blocks	11	12	13	14	15
# states	824073141	12470162233	202976401213	3535017524403	65573803186921
approx.	8.2×10^8	1.3×10^{10}	2.0×10^{11}	3.5×10^{12}	6.6×10^{13}

Blocksworld problems are **PSpace-complete**:
even for a polynomial time algorithm, an exponential amount of memory is needed!

Time and Memory Requirements

Depth	Nodes	Time	Memory	
0	1	1 ms	100	Byte
2	111	0.1 sec	11	Kilo Byte
4	11.111	11 sec	1	Mega Byte
6	10^6	18 min	111	Mega Byte
8	10^8	31 h	11	Giga Byte
10	10^{10}	128 days	1	Tera Byte
12	10^{12}	35 years	111	Tera Byte
14	10^{14}	3500 years	11.111	Tera Byte

Breadth-first search with branching factor $b = 10$, 1000 nodes/sec, 100 bytes/node $\hookrightarrow$ Memory requirements are the bigger problem!

Evaluation of DFS and BFS

- **Soundness:** A node s is only expanded to such a node s' where (s, s') is an arc in the state space (application of a legal operator whose preconditions are fulfilled in s)
- **Termination:** For finite sets of states guaranteed.
- **Completeness:** If a finite length solution exists.
- **Optimality:** Depth-first no, breadth-first yes
- worst case $O(b^d)$ for both, average case better for depth-first
  $\hookrightarrow$ If you know that there exist many solutions, that the average solution length is rather short and if the branching factor is rather high, use depth-first search, if you are not interested in the optimal but just in some admissible solution.
- Prolog is based on a depth-first search-strategy.
- Typical planning algorithms are depth-first.

Uniform Cost Search

- Variation of breadth-first search for operators with different costs.
- Path-cost function $g(n)$: summation of all costs on a path from the root node to the current node n.
  Remark: Please note that $g(n)$ denotes the accumulated costs for one specific path from the root to node n: $\sum_{i=0}^{n} c(i, i+1)$ if nodes i and $i+1$ are on the selected path.
- Costs must be positive, such that $g(n) < g(successor(n))$.
  Remark: This restriction is stronger then necessary. To omit non-termination when searching for an optimal solution it is enough to forbid *negative cycles*.
- Always sort the paths in ascending order of costs.
- If all operators have equal costs, uniform cost search behaves exactly like breadth-first search.
- Uniform cost search is closely related to *branch-and-bound algorithms* (cf. operations research).

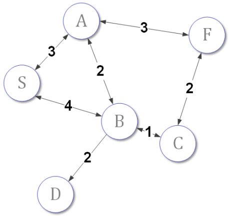
Uniform Cost Algorithm

To conduct a uniform cost search,

- Form a one-element queue consisting of a zero-length path that contains only the root node.
- Until the first path in the queue terminates at the goal node or the queue is empty,
    - Remove the first path from the queue; create new paths by extending the first path to all neighbors of the terminal node.
    - Reject all new paths with loops.
    - Add the new paths, if any, to the queue and *sort* the queue with respect to costs.
- If the goal node is found, announce success; otherwise announce failure.

Uniform Cost Example

(omitting cycles)
((S).0)
((S A).3 (S B).4)
((S A B).5 (S A F).6 (S B).4)
sort
((S B).4 (S A B).5 (S A F).6)
((S B A).6 (S B C).5 (S B D).6 (S A B).5 (S A F).6)
sort
((S A B).5 (S B C).5 (S A F).6 (S B A).6 (S B D).6)
((S A B C).6 (S A B D).7 (S B C).5 (S A F).6 (S B A).6 (S B D).6)
sort
((S B C).5 (S A B C).6 (S A F).6 (S B A).6 (S B D).6 (S A B D).7)
((S B C F).7 (S A B C).6 (S A F).6 (S B A).6 (S B D).6 (S A B D).7)



Uniform Cost Example cont.

((S B C F).7 (S A B C).6 (S A F).6 (S B A).6 (S B D).6 (S A B D).7)
sort
((S A B C).6 (S A F).6 (S B A).6 (S B D).6 (S A B D).7 (S B C F).7)
((S A B C F).8 (S A F).6 (S B A).6 (S B D).6 (S A B D).7 (S B C F).7)
sort
((S A F).6 (S B A).6 (S B D).6 (S A B D).7 (S B C F).7 (S A B C F).8)

Note:

- Termination if first path in the queue (i.e. shortest path) is solution, only then it is guaranteed that the found solution is optimal!
- A more efficient realisation of UCS would override the alphabetical order of terminal nodes when the terminal node is the goal state (of course while keeping the order wrt costs!)

Further Search Algorithms

- **Depth-limited search:**
  Impose a cut-off (e.g. n for searching a path of length $n-1$), expand nodes with max. depth first until cut-off depth is reached (LIFO strategy, since variation of depth-first search).
- **Bidirectional search:**
  forward search from initial state & backward search from goal state, stop when the two searches meet. Average effort $O(b^{\frac{d}{2}})$ if testing whether the search fronts intersect has constant effort $O(1)$.
- In AI, the problem graph is typically not known. If the graph is known, to find *all* optimal paths in a graph with labeled arcs, **standard graph algorithms** can be used. E.g., the Dijkstra algorithm, solving the single source shortest paths problem (calculating the minimal spanning tree of a graph).