Lecture 14: Analytical Learning

Cognitive Systems II - Machine Learning
WS 2005/2006

Part III: Learning Programs and Strategies

Explanation-Based Generalization
Motivation

- Using prior knowledge and deductive reasoning to augment information given by training examples
- → explanation-based learning
- Two variants: background-knowledge is or is not complete and correct
- Reach more accuracy with less examples!
Inductive Learning

Given:
- Instance space $X$
- Hypothesis space $H$
- Training examples $D$ of some target function $f$.

\[ D = \{ \langle x_1, f(x_1) \rangle, \ldots, \langle x_n, f(x_n) \rangle \} \]

Determine:
- A hypothesis from $H$ consistent with training examples $D$. 
Analytical Learning

Given:
- Instance space $X$
- Hypothesis space $H$
- Training examples $D$ of some target function $f$.

$$D = \{\langle x_1, f(x_1) \rangle, \ldots, \langle x_n, f(x_n) \rangle \}$$

- Domain theory $B$ for explaining training examples

Determine:
- A hypothesis from $H$ consistent with both the training examples $D$ and domain theory $B$.

We say
- $B$ “explains” $\langle x, f(x) \rangle$ if $x + B \vdash f(x)$
- $B$ is “consistent with” $h$ if $B \not\vdash \neg h$
SafeToStack(x,y) Learning Problem

Given:

- Instances: pairs of physical objects
- Hypotheses: Sets of Horn clause rules, e.g.,

  \[ \text{SafeToStack}(x, y) \leftarrow \text{Volume}(x, vx) \land \text{Type}(y, \text{Box}) \]

- Training Examples: typical example is

  \[ \text{SafeToStack}(\text{Obj1}, \text{Obj2}) \]
  \[ \text{On}(\text{Obj1}, \text{Obj2}) \]
  \[ \text{Owner}(\text{Obj1}, \text{Fred}) \]
  \[ \text{Type}(\text{Obj1}, \text{Box}) \]
  \[ \text{Owner}(\text{Obj2}, \text{Louise}) \]
  \[ \text{Type}(\text{Obj2}, \text{Endtable}) \]
  \[ \text{Density}(\text{Obj1}, 0.3) \]
  \[ \text{Color}(\text{Obj1}, \text{Red}) \]
  \[ \text{Material}(\text{Obj1}, \text{Cardbd}) \]

- Domain Theory:

  \[ \text{SafeToStack}(x, y) \leftarrow \neg \text{Fragile}(y) \]
  \[ \text{SafeToStack}(x, y) \leftarrow \text{Lighter}(x, y) \]
  \[ \text{Lighter}(x, y) \leftarrow \text{Wt}(x, wx) \land \text{Wt}(y, wy) \land \text{Less}(wx, wy) \]

  \[ \ldots \]
Determine:

- A hypothesis from $H$ consistent with training examples and domain theory.
Prolog-EBG

Prolog-EBG\( (\text{TargetConcept}, \text{Examples}, \text{DomainTheory}) \)

- \( \text{LearnedRules} ← \{\} \)
- \( \text{Pos} ← \text{the positive examples from Examples} \)
- for each \( \text{PositiveExample} \) in \( \text{Pos} \) that is not covered by \( \text{LearnedRules} \), do

  1. Explain:
     - \( \text{Explanation} ← \text{an explanation (proof) in terms of DomainTheory that PositiveExample satisfies TargetConcept} \)

  2. Analyze:
     - \( \text{SufficientConditions} ← \text{the most general set of features of PositiveExample that satisfy TargetConcept according to Explanation} \).
(for each PositiveExample in Pos that is not covered by LearnedRules, do)

3. Refine:
   LearnedRules ← LearnedRules + NewHornClause, where NewHornClause is of the form
   
   TargetConcept ← SufficientConditions

Return LearnedRules
An Explanation

Explanation:

\[ \text{SafeToStack}(\text{Obj1}, \text{Obj2}) \]

\[ \text{Lighter}(\text{Obj1}, \text{Obj2}) \]

\[ \text{Weight}(\text{Obj1}, 0.6) \]

\[ \text{Weight}(\text{Obj2}, 5) \]

\[ \text{Volume}(\text{Obj1}, 2) \]

\[ \text{Density}(\text{Obj1}, 0.3) \]

\[ \text{Equal}(0.6, 2 \times 0.3) \]

\[ \text{LessThan}(0.6, 5) \]

\[ \text{Type}(\text{Obj2}, \text{Endtable}) \]

Training Example:

- **Obj1**: Cardboard, Red, Fred, Box, Material, Owner, Lecture
- **Obj2**: Endtable, Blue, Louise, Type, Material, Weight, 0.6
Computing Weakest Preimage of $SafeToStack(Obj1, Obj2)$

$SafeToStack(x, y)$

$Lighter(Obj1, Obj2)$

$Weight(Obj1, 0.6)$

$Weight(x, wx)$

$LessThan(0.6, 5)$

$Weight(Obj2, 5)$

$Weight(y, wy)$

$LessThan(wx, wy)$

$Volume(x, vx)$

$Density(Obj1, 0.3)$

$Equal(0.6, 2*0.3)$

$Equal(wx, vx*dx)$

$LessThan(wx, wy)$

$Weight(y, wy)$

$Volume(x, vx)$

$Density(x, dx)$

$Equal(wx, vx*dy)$

$LessThan(wx, 5)$

$Type(Obj2, Endtable)$

$Type(y, Endtable)$
Regression

Regress(\textit{Frontier}, \textit{Rule}, \textit{Expression}, \textit{U_{I,R}})

\textit{Frontier}: the set of expressions to be regressed through \textit{Rule}
\textit{Rule}: a horn clause.
\textit{Expression}: the member of \textit{Frontier} that is inferred by \textit{Rule} in the explanation.
\textit{U_{I,R}}: the substitution that unifies \textit{Rule} to the training example in the explanation

Returns the list of expressions forming the weakest preimage of \textit{Frontier} with respect to \textit{Rule}

1. \textit{U_{E,R}} \leftarrow most general unifier of \textit{Expression} with \textit{Consequent} such that there exists a substitution \textit{S} for which
   \[
   S(U_{E,R}(\textit{Consequent})) = U_{I,R}(\textit{Consequent})
   \]

2. Return \textit{U_{E,R}}(\{\textit{Frontier} - \textit{Consequent} + \textit{Antecedent}\})

Example:

Regress(\{Volume(x,vs), Density(x,dx), Equal(wx,vx*dx),
       Less-Than(wx,wy), Weight(y,wy),
       Weight(z,5) :- Type(z,ENDTABLE),
       Weight(y,wy),
       \{OBJ2/z\})

\textit{Consequent} \leftarrow Weight(z,5)
\textit{Antecedents} \leftarrow Type(z,ENDTABLE)
\textit{U_{E,R}} \leftarrow \{y/z, 5/wy\}, \ (S = \{OBJ2/y\})

\textit{Result} \leftarrow \{Volume(x,vs), Density(x,dx), Equal(wx,vx*dx),
       Less-Than(wx,5), Type(y,ENDTABLE)\}
Regress

\[ \text{Regress}(\text{Frontier, Rule, Literal, } \theta_{hi}) \]

*Frontier*: literals to be regressed

*Rule*: A Horn clause

*Literal*: A literal in Frontier inferred by Rule in the explanation

\( \theta_{hi} \): The substitution that unifies the head of Rule to the corresponding literal in the explanation

Returns the set of literals forming the weakest preimage of Frontier with respect to Rule

- \( \text{head} \leftarrow \text{head of Rule} \)
- \( \text{body} \leftarrow \text{body of Rule} \)
- \( \theta_{hl} \leftarrow \text{the most general unifier of head with Literal such that there exists a substitution } \theta_{li} \text{ for which} \)
  \[ \theta_{li}(\theta_{hl}(\text{head})) = \theta_{hi}(\text{head}) \]
- Return \( \theta_{hl}(\text{Frontier} - \text{head} + \text{body}) \)
Example

Regressing Literals

Example (the bottommost regression step):

\[
\text{Regress} (\text{Frontier, Rule, Literal, } \theta_{hi}) \text{ where}
\]

\[
\begin{align*}
\text{Frontier} &= \{\text{Volume}(x, vs), \text{Density}(x, dx), \text{Eq}(wx, \times(vx, dx)), \\
&\text{LessThan}(wx, wy), \text{Weight}(y, wy)\}\n\]
\[
\text{Rule} = \text{Weight}(z, 5) \leftarrow \text{Type}(z, \text{Endtable})
\]
\[
\text{Literal} = \text{Weight}(y, wy)
\]
\[
\theta_{hi} = \{z/\text{Obj2}\}
\]
\[
\begin{align*}
\text{head} &\leftarrow \text{Weight}(z, 5) \\
\text{body} &\leftarrow \text{Type}(z, \text{Endtable}) \\
\theta_{hi} &\leftarrow \{z/y, wy/5\}, \text{ where } \theta_{li} = \{y/\text{Obj2}\}
\]
\[
\text{Return} \{\text{Volume}(x, vs), \text{Density}(x, dx), \text{Eq}(wx, \times(vx, dx)), \\
\text{LessThan}(wx, 5), \text{Type}(y, \text{Endtable})\}
\]

Inductive Bias
Combining Inductive and Analytical Learning
Case-Based and Analogical Reasoning