Lecture 14: Analytical Learning

Cognitive Systems II - Machine Learning

WS 2005/2006

Part III: Learning Programs and Strategies

Explanation-Based Generalization

Motivation

- Using prior knowledge and deductive reasoning to augment information given by trainings examples
- $\hookrightarrow$ explanation-based learning
- Two variants: background-knowledge is or is not complete and correct
- Reach more accuracy with less examples!

Inductive Learning

Given:

- Instance space X
- Hypothesis space H
- Training examples D of some target function f.

$$D = \{\langle x_1, f(x_1)\rangle, \ldots \langle x_n, f(x_n)\rangle\}$$

Determine:

- A hypothesis from H consistent with training examples D.

Analytical Learning

Given:

- Instance space X
- Hypothesis space H
- Training examples D of some target function f.

$$D = \{\langle x_1, f(x_1)\rangle, \ldots \langle x_n, f(x_n)\rangle\}$$

- *Domain theory* B *for explaining training examples*

Determine:

- A hypothesis from H consistent with both the training examples D and domain theory B.

We say

- B "explains" $\langle x, f(x)\rangle$ if $x + B \vdash f(x)$
- B is "consistent with" h if $B \nvdash \neg h$

SafeToStack(x,y) Learning Problem

Given:

- Instances: pairs of physical objects

  Hypotheses: Sets of Horn clause rules, e.g.,

  $$SafeToStack(x, y) \leftarrow Volume(x, vx) \wedge Type(y, Box)$$

- Training Examples: typical example is

  $SafeToStack(Obj1, Obj2)$

$On(Obj1, Obj2)$	$Owner(Obj1, Fred)$
$Type(Obj1, Box)$	$Owner(Obj2, Louise)$
$Type(Obj2, Endtable)$	$Density(Obj1, 0.3)$
$Color(Obj1, Red)$	$Material(Obj1, Cardbd)$

  $\ldots$

- Domain Theory:

  $SafeToStack(x, y) \leftarrow \neg Fragile(y)$

  $SafeToStack(x, y) \leftarrow Lighter(x, y)$

  $Lighter(x, y) \leftarrow Wt(x, wx) \wedge Wt(y, wy) \wedge Less(wx, wy)$

  $\ldots$

SafeToStack(x,y) cont.

Determine:

- A hypothesis from H consistent with training examples and domain theory.

Prolog-EBG

Prolog-EBG($TargetConcept, Examples, DomainTheory$)

- $LearnedRules \leftarrow \{\}$
- $Pos \leftarrow$ the positive examples from $Examples$
- for each $PositiveExample$ in Pos that is not covered by $LearnedRules$, do

1. Explain:

- $Explanation \leftarrow$ an explanation (proof) in terms of *DomainTheory* that *PositiveExample* satisfies *TargetConcept*

2. Analyze:

- $SufficientConditions \leftarrow$ the most general set of features of *PositiveExample* that satisfy *TargetConcept* according to $Explanation$.

Prolog-EBG cont.

- (for each $PositiveExample$ in Pos that is not covered by $LearnedRules$, do)

  3. Refine:

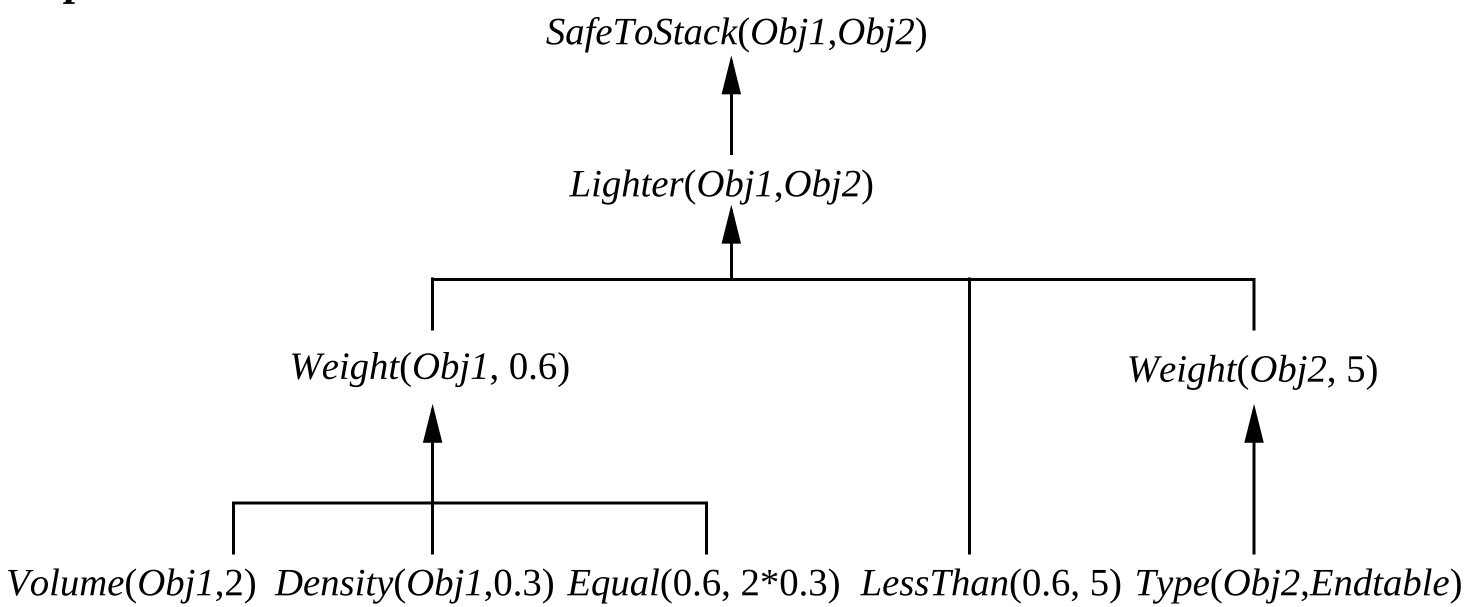
  - $LearnedRules \leftarrow LearnedRules + NewHornClause$, where *NewHornClause* is of the form

$$TargetConcept \leftarrow SufficientConditions$$

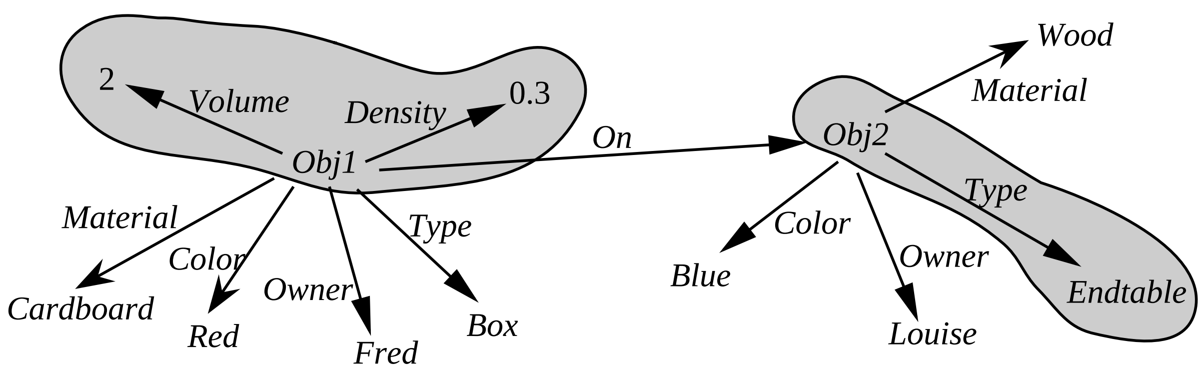
- Return $LearnedRules$

An Explanation

Explanation:

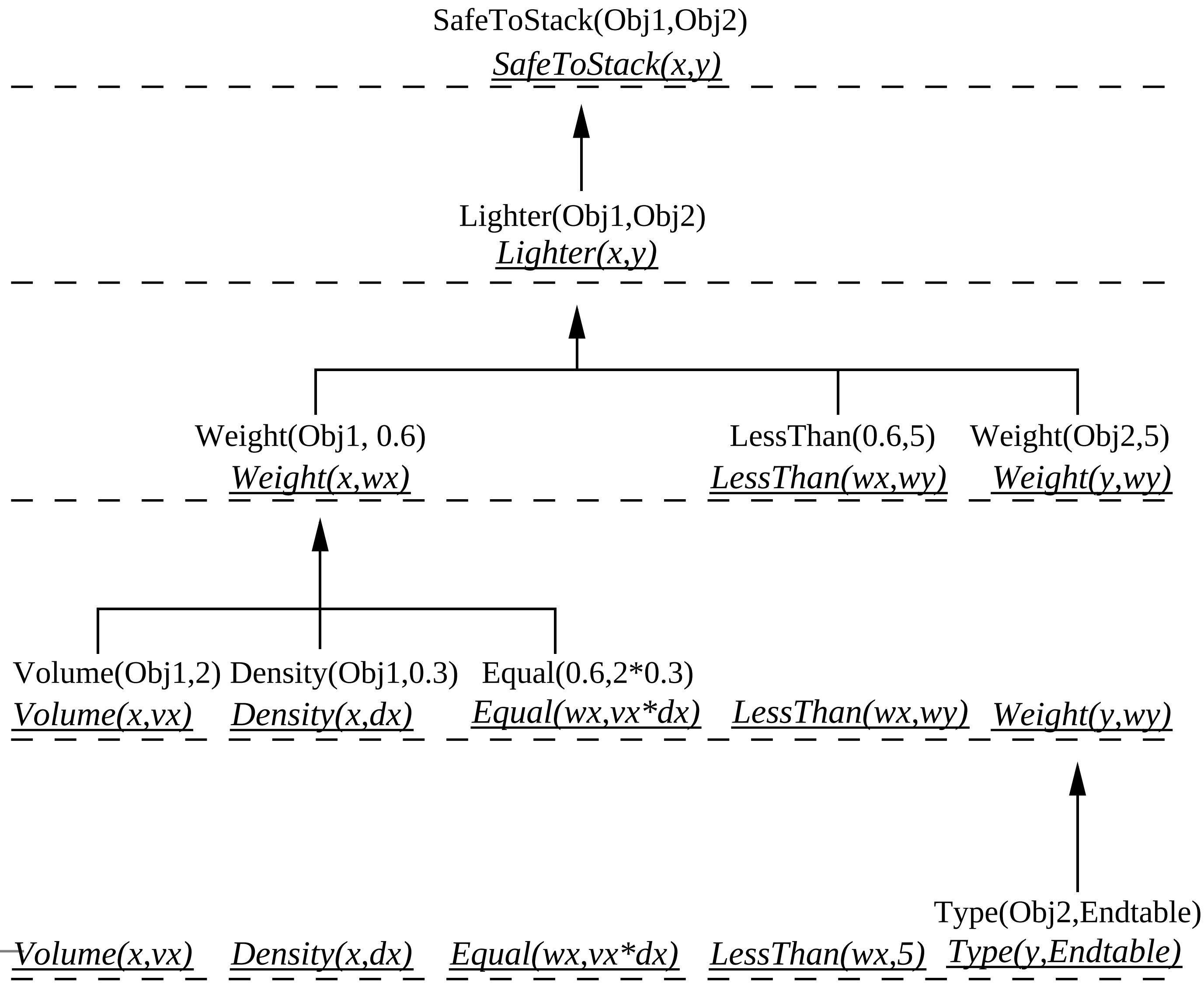


Training Example:



Computing Weakest Preimage

Computing Weakest Preimage of $SafeToStack(Obj1, Obj2)$



Regression

Regress(*Frontier, Rule, Expression,* $U_{I,R}$)

Frontier: the set of expressions to be regressed through *Rule*
Rule: a horn clause.
Expression: the member of *Frontier* that is inferred by *Rule* in the explanation.
$U_{I,R}$: the substitution that unifies *Rule* to the training example in the explanation

Returns the list of expressions forming the weakest preimage of *Frontier* with respect to *Rule*

let Consequent ← *Rule* consequent

let Antecedents ← *Rule* antecedents

1. $U_{E,R}$ ← most general unifier of *Expression* with Consequent
   such that there exists a substitution S for which

   $S(U_{E,R}(\text{Consequent})) = U_{I,R}(\text{Consequent})$

2. Return $U_{E,R}$({*Frontier* -Consequent+Antecedent})

Example:

Regress({Volume(x,vs), Density(x,dx), Equal(wx,vx*dx),
Less-Than(wx,wy), Weight(y,wy)},
Weight(z,5) :- Type(z,ENDTABLE),
Weight(y,wy),
{OBJ2/z})

Consequent ← Weight(z,5)

Antecedents ← Type(z,ENDTABLE)

$U_{E,R}$ ← {y/z, 5/wy}, (S = {OBJ2/y})

Result ← {Volume(x,vs), Density(x,dx), Equal(wx,vx*dx),
Less-Than(wx,5), Type(y,ENDTABLE)}

Regress

Regress(*Frontier, Rule, Literal,* θ_{hi})

Frontier: literals to be regressed

Rule: A Horn clause

Literal: A literal in Frontier inferred by Rule in the explanation

θ_{hi}*: The substitution that unifies the head of Rule to the corresponding literal in the explanation*

Returns the set of literals forming the weakest preimage of Frontier with respect to Rule

- $head \leftarrow head$ of *Rule*
- $body \leftarrow body$ of *Rule*
- $\theta_{hl} \leftarrow$ the most general unifier of $head$ with *Literal* such that there exists a substitution θ_{li} for which

$$\theta_{li}(\theta_{hl}(head)) = \theta_{hi}(head)$$

- Return $\theta_{hl}(Frontier - head + body)$

Example

Regressing Literals

Example (the bottommost regression step):

Regress(*Frontier, Rule, Literal,* θ_{hi}) where

Frontier $= \{Volume(x, vs), Density(x, dx), Eq(wx, times(vx,dx)),$ $LessThan(wx,wy)$, $Weight(y,wy)\}$

Rule $= Weight(z, 5) \leftarrow Type(z, Endtable)$

Literal $= Weight(y, wy)$

$\theta_{hi} = \{z/Obj2\}$

- $head \leftarrow Weight(z, 5)$
- $body \leftarrow Type(z, Endtable)$
- $\theta_{hl} \leftarrow \{z/y, wy/5\}$, where $\theta_{li} = \{y/Obj2\}$
- Return $\{Volume(x, vs), Density(x, dx), Eq(wx, times(vx, dx)),$ $LessThan(wx, 5), Type(y, Endtable)\}$

Inductive Bias

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Case-Based and Analogical Reasoning