Lecture 6: Inductive Logic Programming

Cognitive Systems II - Machine Learning

Part II: Special Aspects of Concept Learning

FOIL, Inverted Resolution, Sequential Covering

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Motivation

- it is useful to learn the target function as a set of if-then-rules
  - one of the most expressive and human readable representations
  - e.g. decision trees
- **Inductive Logic Programming (ILP):**
  - rules are learned directly
  - designed to learn *first-order rules* (i.e. including variables)
  - *sequential covering* to incrementally grow the final set of rules
- PROLOG programs are sets of first-order rules

$\Rightarrow$ a general-purpose method capable of learning such rule sets may be viewed as an algorithm for automatically inferring PROLOG programs

Examples

- Propositional Logic:

```
IF Humidity=normal AND Outlook=sunny THEN PlayTennis=y
IF Humidity=normal AND Temperature=mild AND Wind=weak

playTennis :- humidity(normal), outlook(sunny).
playTennis :- humidity(normal), temperature(mild), win
```

- First Order Logic:

```
IF Parent(x,y) THEN Ancestor(x,y)
IF Parent(x,z) AND Ancestor(z,y) THEN Ancestor(x,y)

ancestor(x,y) :- parent(x,y).
ancestor(x,y) :- parent(x,z), ancestor(z,y).
```

Sequential Covering

- **basic strategy**: learn one rule, remove the data it covers, then iterate this process
- one of the most widespread approaches to learn a disjunctive set of rules (each rule itself is conjunctive)
- subroutine **LEARN-ONE-RULE**
  - accepts a set of positive and negative examples as input and outputs a single rule that covers many of the positive and few of the negative examples
  - **high accuracy**: predictions should be correct
  - **low coverage**: not neccessarily predictions for each example
- performs a greedy search without backtracking

  $\Rightarrow$ no guarantee to find the smallest or best set of rules

Sequential Covering Algorithm

SEQUENTIAL-COVERING($Target\_attribute, Attributes, Examples, Threshold$)

- $Learned\_Rules \leftarrow \{\}$
- $Rule \leftarrow$ **LEARN-ONE-RULE**($Target\_attribute, Attributes, Examples$)
- While **PERFORMANCE**($Rule, Examples$) $> Threshold$, Do
  - $Learned\_rules \leftarrow Learned\_rules + Rule$
  - $Examples \leftarrow Examples - \{$ examples correctly classified by $Rule\}$
  - $Rule \leftarrow$ **LEARN-ONE-RULE**($Target\_attribute, Attributes, Examples$)
- $Learned\_rules \leftarrow$ sort $Learned\_rules$ accord to **PERFORMANCE** over $Examples$
- return $Learned\_rules$

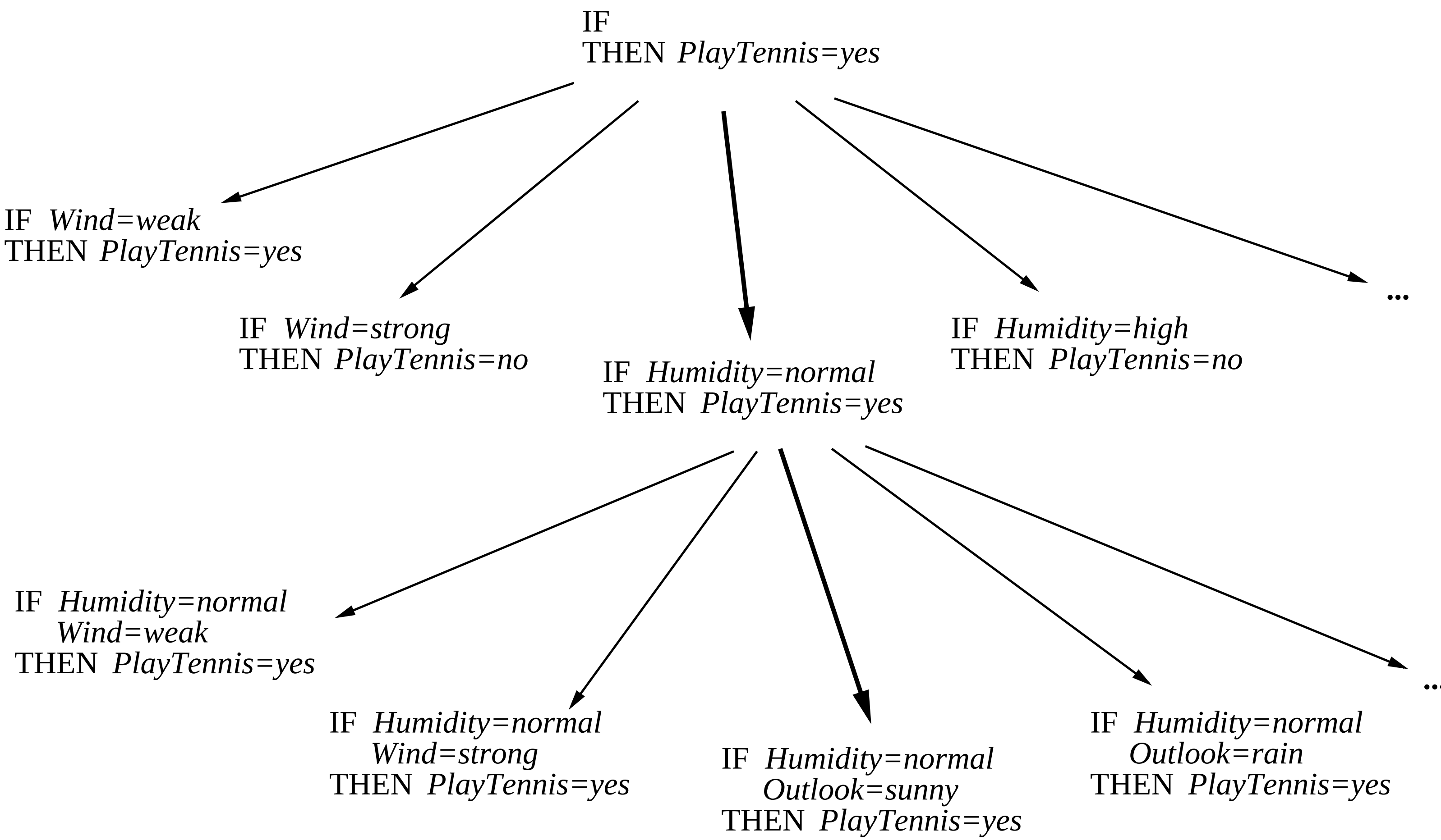
General to Specific Beam Search

- **question**: How shall LEARN-ONE-RULE be designed to to meet the needs of the sequential covering algorithm?

- organize the search through H analoguous to ID3
  - **but** follow only the most promising branch in the tree at each step
  - begin by considering the most general rule precondition (i.e. empty test)
  - then greedily add the attribute test that most improves rule performance over the training examples

  - unlike to ID3, this implementation follows only a single descendant at each search step rather than growing a subtree that covers all possible values of the selected attribute

General to Specific Beam Search



General to Specific Beam Search

- **so far** a local greedy search (analoguous to hill-climbing) is employed
  - danger of suboptimal results
  - susceptible to the typical hill-climbing problems

$\Rightarrow$ extension to **beam search**

  - algorithm maintains a list of the k best candidates at each step
  - at each step, descendants are generated for each of the k candidates and the resulting set is again reduced to the k most promising candidates

LEARN-ONE-RULE

LEARN-ONE-RULE($Target\_attribute, Attributes, Examples, k$)
Returns a single rule that covers some of the Examples. Conducts a general to specific greedy beam search for the best rule, guided by the PERFORMANCE metric.

- Initialize $Best\_hypothesis$ to the most general hyothesis $\emptyset$
- Initialize $Candidate\_hypotheses$ to the set $\{Best\_hypothesis\}$
- While $Candidate\_hypotheses$ is not empty, Do
  - *1. Generate the next more specific candidate_hypotheses*
    - $New\_Candidate\_hypotheses \leftarrow$ new generated and specialized candidates
  - *2. Update Best_hypotheses*
    - Select hypothesis h from $New\_candidate\_hypotheses$ with best PERFORMANCE over Examples
    - IF PERFORMANCE of $h >$ PERFORMANCE of $Best\_hypothesis$ THEN set h as new $Best\_hypothesis$
  - *3. Update Candidate_hypotheses*
    - $Candidate\_hypotheses \leftarrow$ the k best members of $New\_Candidate\_hypotheses$

LEARN-ONE-RULE cont.

- Return a rule of the form

  "IF $Best\_hypothesis$ THEN $prediction$"

  where $prediction$ is the most frequent value of $Target\_attribute$ among those $Examples$ that match $Best\_hypothesis$.

Example: Learn One Rule

- Learn *one* rule that covers a certain amount of positive examples
- with high accuracy
- remove the covered positive examples

Example:

(s1)	Sky = sunny	(s2)	Sky = rainy
(a1)	AirTemp = warm	(a2)	AirTemp = cold
(h1)	Humidity = normal	(h2)	Humidity = high
(w1)	Water = warm	(w2)	Water = cool

Example cont.

- Assume $k = 4$
- Current most specific hypotheses: s1, s2, a1, a2, h1, h2, w1, w2
- Assume best possible hypotheses wrt performance P: s1, a2, h1, w1
- Generate new candidate hypotheses, e.g. by specializing s1:

s1 & s1 (dublicate)

s1 & s2 (inconsistent)

s1 & a1

s1 & a2

s1 & h1

Performance Measures

- Relative Frequency (numbers of correctly classified examples by all examples)

$$\frac{n_c}{n}$$

- Entropy (S as set of examples that match precondition, p_i proportion of examples from S for which the target function takes the i-th value

$$-Entropy(S) = \sum_{i=1}^{c} p_i log_2 p_i$$

Sequential vs. Simultaneous Covering

- **sequential covering:**
  - learn just one rule at a time, remove the covered examples and repeat the process on the remaining examples
  - many search steps, making independent decisions to select each precondition for each rule
- **simultaneous covering:**
  - ID3 learns the entire set of disjunctive rules simultaneously as part of a single search for a decision tree
  - fewer search steps, because each choice influences the preconditions of all rules

$\Rightarrow$ Choice depends of how much data is available

- plentiful $\rightarrow$ sequential covering (more steps supported)
- scarce $\rightarrow$ simultaneous covering (decision sharing effective)

Differences in Search

- **generate-then-test:**
  - search through all syntactically legal hypotheses
  - generation of the successor hypotheses is only based on the syntax of the hypothesis representation
  - training data is considered after generation to choose among the candidate hypotheses
  - each training example is considered several times

  $\Rightarrow$ impact of noisy data is minimized

- **example driven:**
  - individual training examples constrain the generation of hypotheses
  - e.g. FIND-S, CANDIDATE ELIMINATION

  $\Rightarrow$ search can easily be misled

Learning First-Order Rules

- propositional expressions do not contain variables and are therefore less expressive than first-order expressions
- no general way to describe essential relations among the values of attributes
- now we consider learning first-order rules (Horn Theories)
  - a Horn clause is a clause containing at most one positive literal
  - expression of the form:

    $$H \vee \neg L_1 \vee \neg L_2 \vee \ldots \vee \neg L_n$$
    $$\Longleftrightarrow H \leftarrow (L_1 \wedge L_2 \wedge \ldots \wedge L_n)$$
    $$\Longleftrightarrow \text{IF } (L_1 \wedge L_2 \wedge \ldots \wedge L_n) \text{ THEN } H$$
  - FOL terminology see CogSysI

FOIL (Quinlan, 1990)

- natural extension of SEQUENTIAL-COVERING and LEARN-ONE-RULE
- outputs sets of first-order rules similar to Horn Clauses with two exceptions
  1. **more restriced**, because literals are not permitted to contain function symbols
  2. **more expressive**, because literals in the body can be negated
- differences between FOIL and earlier algorithms:
  - seeks only rules that predict when the target literal is $True$
  - conducts a simple hill-climbing search instead of beam seach

FOIL algorithm

FOIL($Target\_predicate, Predicates, Examples$)

- $Pos \leftarrow$ those $Examples$ for which the $Target\_predicate$ is $True$
- $Neg \leftarrow$ those $Examples$ for which the $Target\_predicate$ is $False$
- $Learned\_rules \leftarrow \{\}$
- while Pos, Do
  - $NewRule \leftarrow$ the rule that predicts $Target\_predicate$ with no precondition
  - $NewRuleNeg \leftarrow Neg$
  - while $NewRuleNeg$, Do
    - $Candidate\_literals \leftarrow$ generate new literals for $NewRule$, based on $Predicates$
    - $Best\_literal \leftarrow \max_{L \in Candidate\_literals} FoilGain(L, NewRule)$
    - add $Best\_literal$ to preconditions of $NewRule$
    - $NewRuleNeg \leftarrow$ subset of $NewRuleNeg$ that satisfies $NewRule$ preconditions
  - $Learned\_rules \leftarrow Learned\_rules + NewRule$
  - $Pos \leftarrow Pos -$ { members of Pos covered by $NewRule$}
- Return $Learned\_rules$

FOIL Hypothesis Space

- **outer loop (set of rules):**
  - specific-to-general search
  - initially, there are no rules, so that each example will be classified negative (most specific)
  - each new rule raises the number of examples classified as positive (more general)
  - disjunctive connection of rules
- **inner loop (preconditions for one rule):**
  - general-to-specific search
  - initially, there are no preconditions, so that each example satisfies the rule (most general)
  - each new precondition raises the number of examples classified as negative (more specific)
  - conjunctive connection of preconditions

Generating Candidate Specializations

- current rule:

  $P(x_1, x_2, ..., x_k) \leftarrow L_1...L_n$ where

  $L_1...L_n$ are the preconditions and

  $P(x_1, x_2, ..., x_k)$ is the head of the rule

- FOIL generates candidate specializations by considering new literals L_{n+1} that fit one of the following forms:
  - $Q(v_1, ..., v_r)$ where $Q \in Predicates$ and the v_i are new or already present variables (at least one v_i must already be present)
  - $Equal(x_j, x_k)$ where x_j and x_k are already present in the rule
  - the negation of either of the above forms

FOIL Example

GrandDaughter(x,y) $\leftarrow$
Cadidate additions to precondition:
Equal(x,y), Female(x), Female(y), Father(x,y), Father(y,x), Father(x,z), Father(z,x), Father(z,y), and the negations to these literals
Assume greedy selection of Father(y,z):
GrandDaughter(x,y) $leftarrow$ Father(y,z)
Candidate additions:
the ones from above and Female(z), Equal(z,y), Father(z,w), Father(w,z), and their negations
Learned Rule:
GrandDaughter(x,y) $leftarrow$ Father(y,z) $\wedge$ Father(z,x) $\wedge$ Female(y)

FOIL Gain

Learning Recursive Rule Sets

Induction as Inverted Deduction

- **observation:** induction is just the inverse of deduction
- in general, machine learning involves building theories that explain the observed data
- Given some data D and some background knowledge B, learning can be described as generating a hypothesis h that, together with B, explains D.

$$(\forall < x_i, f(x_i) > \in D)(B \wedge h \wedge x_i) \vdash f(x_i)$$

- the above equation casts the learning problem in the framework of deductive inference and formal logic

Induction as Inverted Deduction

- **features of inverted deduction:**
  - subsumes the common definition of learning as finding some general concept
  - background knowledge allows a more rich definition of when a hypothesis h is said to “fit” the data
- **practical difficulties:**
  - noisy data makes the logical framework completely lose the ability to distinguish between truth and falsehood
  - search is intractable
  - background knowledge often increases the complexity of H

Inverting Resolution

- **resolution** is a general method for automated deduction
- complete and sound method for deductive inference
- see CogSys1
- **Inverse Resolution Operator (propositional form):**
  1. Given initial clause C_1 and C, find a literal L that occurs in C_1 but not in clause C.
  2. Form the second clause C_2 by including the following literals

     $$C_2 = (C - (C_1 - \{L\})) \cup \{\neg L\}$$
- inverse resolution is not deterministic

Inverting Resolution

C_2: KnowMaterial v ~Study

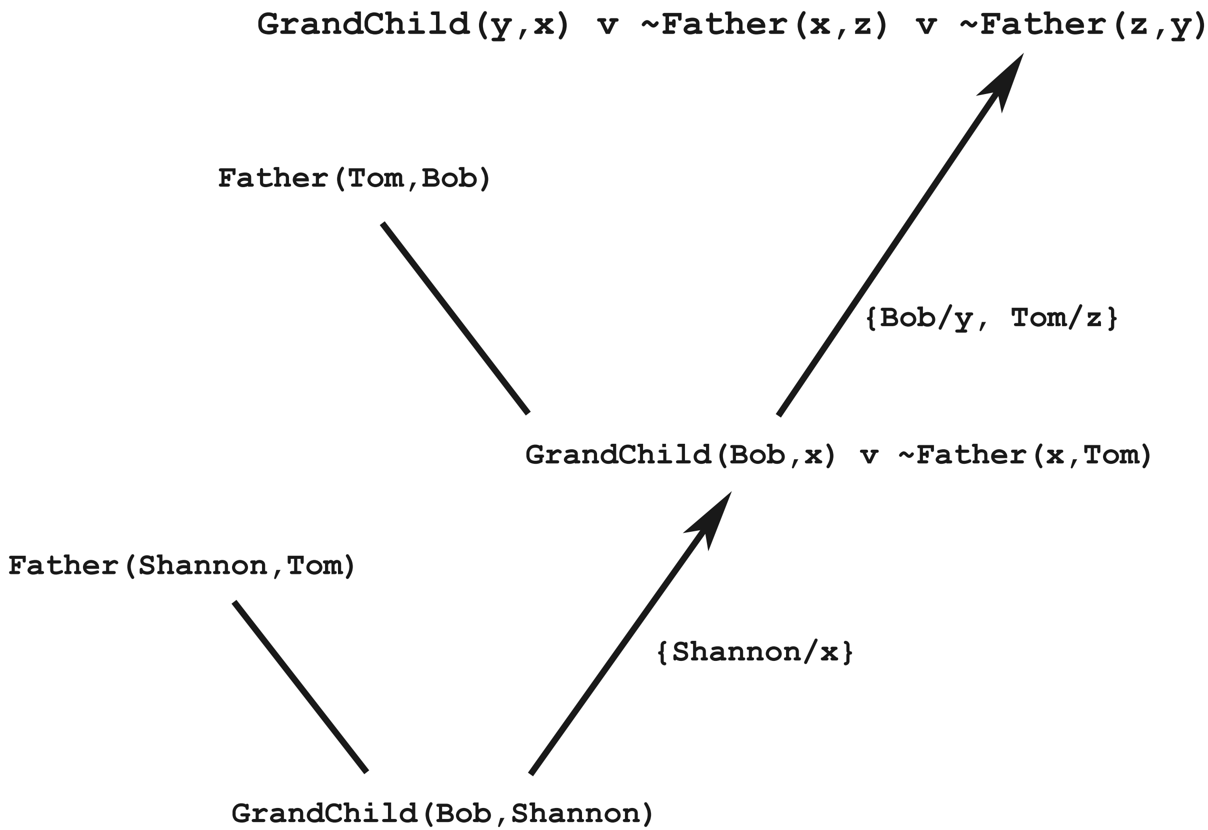
C_1: PassExam v ~KnowMaterial

C: PassExam v ~Study

Inverting Resolution

- **Inverse Resolution Operator (first-order form):**
  - resolution rule:
    1. Find a literal L_1 from clause C_1, literal L_2 from clause C_2, and substitution θ such that $L_1\theta = \neg L_2\theta$
    2. Form the resolvent C by including all literals from $C_1\theta$ and $C_2\theta$, except for $L_1\theta$ and $\neg L_2\theta$. That is,
       $$C = (C_1 - \{L_1\})\theta \cup (C_2 - \{L_2\})\theta$$
  - analytical derivation of the inverse resolution rule:
    $$C = (C_1 - \{L_1\})\theta_1 \cup (C_2 - \{L_2\})\theta_2 \text{ where } \theta = \theta_1\theta_2$$
    $$C - (C_1 - \{L_1\})\theta_1 = (C_2 - \{L_2\})\theta_2 \text{ where } L_2 = \neg L_1\theta_1\theta_2^{-1}$$
    $$\Rightarrow \quad C_2 = (C - (C_1 - \{L_1\})\theta_1)\theta_2^{-1} \cup \{\neg L_1\theta_1\theta_2^{-1}\}$$

Inverting Resolution



$$D = \{GrandChild(Bob, Shannon)\}$$

$$B = \{Father(Shannon, Tom), Father(Tom, Bob)\}$$

PROGOL

Generalization, θ-Subsumption, Entailment

- interesting to consider the relationship between the $more\_general\_than$ relation and inverse entailment

- $more\_general\_than$: $h_j \geq_g h_k$ iff $(\forall x \in X)[h_k(x) \rightarrow h_j(x)]$. A hypothesis can also be expressed as $c(x) \leftarrow h(x)$.

- $\theta - subsumption$: Consider two clauses C_j and C_k, both of the form $H \vee L_1 \vee ... \vee L_n$, where H is a positive literal and the L_i are arbitrary literals. Clause C_j is said to $\theta - subsume$ clause C_k iff $(\exists\theta)[C_j\theta \subseteq C_k]$.

- $Entailment$: Consider two clauses C_j and C_k. Clause C_j is said to $entail$ clause C_k (written $C_j \vdash C_k$) iff C_k follows deductively from C_j.

Generalization, θ-Subsumption, Entailment

- if $h_1 \geq_g h_2$ then $C_1 : c(x) \leftarrow h_1(x)$ θ-subsumes $C_2 : c(x) \leftarrow h_2(x)$
- furthermore, θ-subsumption can hold even when the clauses have different heads

  $A : Mother(x, y) \leftarrow Father(x, z) \wedge Spouse(z, y)$

  $B : Mother(x, L) \leftarrow Father(x, B) \wedge Spouse(B, y) \wedge Female(x)$

  $A\theta \subseteq B$ if we choose $\theta = \{y/L, z/B\}$
- θ-subsumption is a special case of entailment

  $A : Elephant(father\_of(x)) \leftarrow Elephant(x)$

  $B : Elephant(father\_of(father\_of(y))) \leftarrow Elephant(y)$

  $A \vdash B$, but $\neg\exists\theta[A\theta \subseteq B]$

Generalization, θ-Subsumption, Entailment

- Generalization is a special case of θ-Subsumption
- θ-Subsumption is a special case of entailment
- In its most general form, inverse entailment produces intractable searches
- θ-Subsumption provides a convenient notion that lies midway between generalization and entailment!

Summary

- learns sets of first-order rules directly
- sequential covering algorithms learn just one rule at a time and perform many search steps
- hence, applicable if data is plentiful
- **FOIL** is a sequential covering algorithm
- a specific-to-general search is performed to form the result set
- a general-to-specific search is performed to form each new rule
- Induction can be viewed as the inverse of deduction
- hence, an inverse resolution operator can be found