Do People Prefer Irrational Ratios? A New Look at the Golden Section

Peter Großmann (peter-grossmann@stud.uni-bamberg.de)
Student of Applied Computer Science, University of Bamberg

Tobias Kärner (tobias-ulrich.kaerner@stud.uni-bamberg.de)
Student of Education of Business and Information Systems, University of Bamberg

Christian Maier (christian-engelbert.maier@stud.uni-bamberg.de)
Student of Economic Computer Science, University of Bamberg

Michael Räther (michael.raether@stud.uni-bamberg.de)
Student of Economic Computer Science, University of Bamberg

Stefan Reisinger (stefan-rainer.reisinger@stud.uni-bamberg.de)
Student of Applied Computer Science, University of Bamberg

Christoph Sonnenberg (christoph.sonnenberg@stud.uni-bamberg.de)
Student of Psychology, University of Bamberg

Florian Spindler (florian-gertwald1.spindler@stud.uni-bamberg.de)
Student of Applied Computer Science, University of Bamberg

BjörnChristopherWitte (bjoern-christopher.witte@stud.uni-bamberg.de)
Student of Macroeconomics, University of Bamberg

Olga Yanenko (olga.yanenko@stud.uni-bamberg.de)
Student of Applied Computer Science, University of Bamberg

Klaus Stein(klaus.stein@wiai.uni-bamberg.de)
Laboratory for Semantic Information Technology, University of Bamberg

Ute Schmid (ute.schmid@uni-bamberg.de)
Faculty Information Systems and Applied Computer Science, University of Bamberg
Feldkirchenstrasse 28, 96045 Bamberg, Germany1

Abstract

The *Golden Section* is proposed to be the preferred ratio in many domains. Starting with Fechner, there were several empirical studies concerning the preference of the *Golden Section* with inconclusive results. Our main concern is that the *Golden Section* can neither perceived no produced exactly since it is an irrational number. Therefore, we propose to give a closer look to preferences for ratios and not to restrict research to the *Golden Section*. Alternative hypothesis concerning the aesthetics of ratios might be that people prefer ratios in an area near or surrounding the *Golden Section* or that people prefer irrational ratios over rational ones. We conducted an online experiment where the participants had to select aesthetically pleasing sections from panorama pictures in different ratios. In addition to the one dimensional stimuli, in a second trial Fechner rectangles were presented to subjects using the same ratios. Results differ strongly for the one-dimensional ([0.66; 0.72]) and two-dimensional (1.0) stimuli.

1 Introduction

The golden section is probably one of the most mysterious and controversial mathematical figures. It is famous for being the most irrational number measured by the ease of representing it by the fraction of two integers. Interestingly, nature seems to take advantage of the golden section's irrationality. The

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positioning of leaves follows the golden angle guaranteeing optimal isolation, to give an example. Still experts debate whether human beings might have an innate preference for the golden section, too. Green (1995) provides a comprehensive overview of fundamental research projects on this topic. An intrinsic problem to all studies remains that the Golden Section itself can neither be perceived nor displayed due to its high degree of irrationality. Therefore, previous results are open to alternative explanations: First, humans might prefer ratios in the area around the Golden Section but not necessarily the Golden Section itself. Second, humans might prefer ratios with a higher degree of irrationality over simple ratios and not the Golden Section specifically. Our experiment represents a slightly alternative approach to the research problem. While other studies concentrate just on the Golden Section, our study focuses on irrational numbers in general. We assume that the irrationality of a ratio might be responsible for its subjective attractiveness, apart from its distance from the golden one. Paired comparisons within a web-experiment are conducted to test the hypothesis. Probands made half of the comparisons between panorama pictures which were split in variable ratios either by sun or by a lighthouse. The other half of the comparisons took place between pairs of rectangles. The intuition here was to represent different ratios in one as well as in two dimensions. Besides, we applied abstract as well as real-world motives to test for possible differences. Finally, we assumed that security-seeking characters might prefer symmetries. In order to control for this factor every candidate was assessed by means of a personality test.

The results of our study might be useful for the development of picture editing programs inter alia. The program could automatically propose clippings e.g. of a panorama picture, such that certain objects split the new image in ways that appears harmonic to the user.

This article is organized as follows: Chapter two resumes the current state of research on the subject. Chapter three reports the experiment including materials, methods and results. Chapter four summarizes our main findings and reveals needs for future research.

2 Aesthetics of the Golden Section

The following section refers to a meta analysis about the psychological research at the Golden Section written by Christopher D. Green and called ”All The Glitters: A Review of Psychological Research on the Aesthetics of the Golden Section” (Green, 1995). In a first step the basics of the Golden Section will be described, in a second step selected previous research findings will be discussed with regard to our findings below. A ratio between two lines (x, y) is called Golden Section when these two lines fulfill the following conditions: The ratio of the shorter line-segment (x) to the larger line-segment (y) is equal to the ratio of the larger line-segment (y) to the whole line (x + y), which is the sum of the shorter and the larger line segment. If y is equal one, the value of the Golden Section is an irrational constant in mathematics round about 1.618 and called Phi. The ratio of the Golden Section is not limited to the first dimension in geometry. So, if the two angles of a rectangle are in ratio of Phi, the rectangle is called ”golden rectangle” (see figure 1).

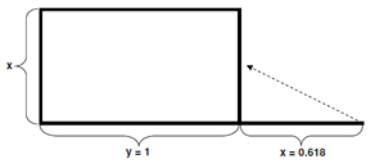

Figure 1: Golden Rectangle

Apart from the one dimensional line and the two dimensional rectangle, the Golden Section also occurs in other geometrical structures, which won’t be discussed in this paper yet.

The Golden Section also appears in elemental biological topologies. For example the shell of the nautilus is grown like a logarithmic spiral, which can be constructed by golden rectangles in a geometrical way. Summarizing the described mathematical and natural phenomenon which ”*holds the key to the secret of beauty*” and has been researched in previous investigations i.e. by psychologists and mathematicians will be the basis for the article at hand.

In this context the further passages refer to selected existing psychological studies (Green, 1995) about the Golden Section to constitute different approaches and their particular advantages and disadvantages.

The first researcher who studied the aesthetic implications of the Golden Section was the German philosopher and physicist Gustav Theodor Fechner who published the results of his experimental investigations in the year 1871 (Fechner, 1978). He developed three methods of investigating the effects of the Golden Section. First the *method of choice*, where the test persons choose an object out of a number of possible objects that they like most (see figure 2 for a sample of Fechners rectangles).

Figure 2: Examples of Fechners Rectangles

Secondly he developed the *method of production*. Here the subjects create a geometrical object (e.g. drawing triangles or rectangles, dividing a line in two parts) in proportions they most agree with. Last but not least the third approach is called *method of use* where pre-existing objects are examined (e. g. proportions of famous paintings frames). Fechner tested 347 people by presenting them a set of ten rectangles and found out, that about 36 percent preferred the one in the Golden Section. But he couldn’t explain this phenomenon scientifically.

Several authors criticized Fechners test arrangement because the composition of the presented rectangles could have advantaged the selection of the medial one, which was the ”golden” (”trend to the mean” - phenomenon). The other points of critic are that the subjects were not randomized selected and could have been influenced in their decisions by knowing Fechners hypothesis. Two of the critics were Haines and Davies (1904), they presented the rectangles to their subjects one after the other at short intervals and asked them to accept or reject intuitionally, avoiding the tendency to the middle. In result the golden rectangle had no effect to the individual choice. In 1931 C. D. Weber, a psychologist, as one of the first who used the method of *paired comparisons* attempted finding out the most preferred rectangle of 68 test persons. All in all 44 percent of the subjects had chosen the golden one, root three or 1.871:1. Another researcher was Farnsworth (1932) who used nine grey rectangles in different orientations (horizontal, vertical) in a first step, whereby the golden rectangle in the vertical position was preferred most. In a second step he presented black rectangles on a white background. The result was no strong preference of the golden one. So Farnsworth concluded that the Golden Section must be ”*a function of size and color of the rectangle*”. All the described experiments refer to Fechners method of choice. One investigator using the *method of production* is called Davis (1933). He let his 310 probands draw their favored rectangle for the first time and repeated this procedure a second time. The result was that the golden rectangle was drawn by only three percent of the test persons. The main problem in reference to the *method of production* is the inaccuracy of drawing and measuring the correct ”golden” proportions because of the irrationality of Phi. This problem appears less in the *method of choice* because the measures of the respective rectangles can be chosen unambiguous previously.

Except for geometrical structures other visual objects have been investigated concerning to the effects of the Golden Section. For example Boselie (1992) tested the analysis of Bouleau (1980), who analyzed a painting of Mondrian, in an empirical way. In the context of real artwork there are ”*perceptual dynamics operating*” (e.g. color, geometrically interdependencies). They are possibly superposing the

psychological effects of the Golden Section. So it’s difficult to make a statement if the apperception of non-abstract pictures or paintings is also influenced by this mysterious phenomenon.

Summing up the psychological research about the Golden Section and the influence of them to peoples aesthetics perception there are many different findings beyond sexes, ages or geographical areas. Over one hundred years of scientific research beginning from the early 19th to the 21st century generated miscellaneous results. So it can’t be concluded terminatory if the aesthetic perception is affected by the Golden Section, or generally by the irrationality of proportions, or not. There are many questions unanswered, e.g. the influence of irrationality in non-abstract pictures or three-dimensional arrangements.

3 Experiment: Preference of the Irrational?

This chapter describes the experiment's methods. It includes the material, design and procedure as well as the results.

3.1 Material

Since it is not possible to display or perceive irrational ratios (including the Golden Section) precisely, we decided to contrast ratios with small denominators (rational) with ratios with large denominators (irrational). More precisely we defined those ratios with denominator smaller than 5 as rational ratios and took the following ratios: 0.25; 0.33; 0.4; 0.5; 0.6; 0.66; 0.75. With ratios greater than 25, irrational ratios are meant. The following ratios are used: 0.28; 0.32; 0.39; 0.47; 0.53; 0.618 – Golden Section; 0.68; 0.72. We wanted to evaluate one and two-dimensional ratios and for both, we used paired comparisons because statistical methods are better applicable. Another benefit is that the error of central tendency cannot occur.

3.1.1 Evaluating one-dimensional ratios

As main material we selected a panorama picture displaying a lighthouse (see figure 3) and another displaying a sunset (see figure 4). That is, we selected natural pictures containing one distinct object – oblong in one case, round in the other case. Thereby we could test whether the form of an object has an influence on the preferred ratio. To control the influence of possible distractions in the natural material, additionally we presented "panorama pictures" with artificial stimuli – a longish rectangle similar to the lighthouse (see figure 5) and a circle similar to the sunset (see figure 6).

Figure 3: Panorama picture displaying a lighthouse

Figure 4: Panorama picture displaying a sunset

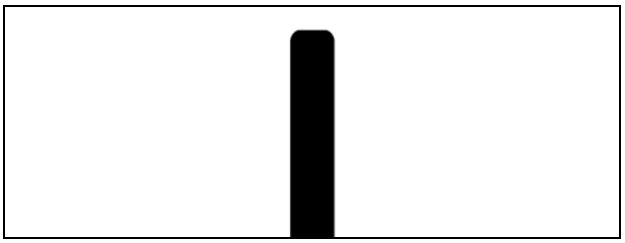

Figure 5: Panorama picture displaying a rectangle similar to the lighthouse

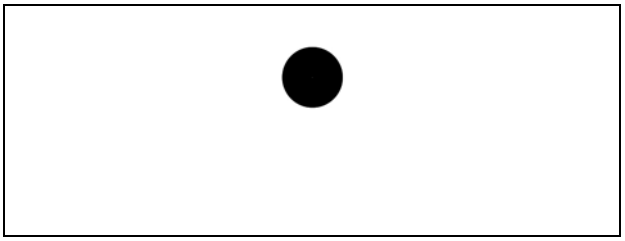

Figure 6: Panorama picture displaying a circle similar to the sunset

We used a between-subjects design, so that the values of the dependent variable for one group of participants were compared with the values for another group.

3.1.2 Evaluating two-dimensional ratios

In the two-dimensional case we used the rectangles of Fechner as mentioned in chapter ”Aesthetics of the Golden Section”. We took the same ratios as in the one-dimensional case plus 1.0 (the square).

3.1.3 Impact of Personality on Evaluation

Furthermore, we assume that there are different personal preferences concerning the symmetry. Therefore we use a kind of personality test in order to check if there is an influence of personality on the selection behavior. It could be possible that people who have a strong need for security prefer symmetrical configurations. Within our project we couldn’t afford a professional personality test, so we created 15 items which should classify our probands. Each proband has to assess if a given statement is suitable for his personality, given a scale of five possible answers from high acceptance to high rejection. All of our 15 items are used to decide if a proband is rather a secure or rather an insecure person. In order to prevent that a test person is just selection one value of our scale, some statements have a

reversed polarity. The following table shows the 15 items which we used in our experiment. The items with reversed polarity are marked with ”reversed”.

1	Before going shopping, I'm almost every time writing a shopping list.
2	I'm very adventurous und enjoy making new experiences. *(reversed)*
3	I try to prevent deadline pressure and fever by making accurate plans.
4	In principle, I'm rather an anxious person.
5	Difficult tasks for which I do not have a solution are taken as a challenge by me. *(reversed)*
6	I try to guarantee that my things stay proper and clean.
7	I prefer quick acting from making wide plans. *(reversed)*
8	It's hard for me to find a suitable topic of conversation in order to meet someone.
9	Even a plurality of interferences can't irritate me. *(reversed)*
10	I try to avoid situations which are not familiar and in which I don't know what to expect.
11	Even without having serious pain, I'm seeing a doctor regularly.
12	I prefer staying in the background at social and public gatherings.
13	I'm having a set of aims and I'm following these aims systematically.
14	There are few things which can make me uneasy very quickly. *(reversed)*
15	I'm feeling uncomfortable if I have to change my plans spontaneously.

Table 1: Items of the personality test

3.2 Design and Procedure

In order to achieve a large attendance, the experimentation was realized as a website. On that account we were able to collect multiple data in a short time.

After a brief description of the experiment we instructed the participant to adjust the resolution of his monitor to the maximum because then the width and height of the displayed pixels should be the same. This was necessary for the two dimensional test, where the rectangles would no longer have the purposed side-to-side ratio. After this introduction we collected some personal data such as age, gender, education, occupation and derivation. This was followed by the personality test, which has the function to check the participant’s need for security. The main part of the experiment contained the one dimensional and the two dimensional test. First pairs of cutouts from one of the panorama views with different ratios according to the position of the object in the image were displayed. Every proband had to deal with one of the four panorama pictures (see figures 3-6). The participant was advised to choose in impulse the one that seems more pleasant to him. The same procedure was used in the rectangle test. To avoid sequence effects we used four different permutations. The first one is a random order of paired comparisons where the pictures of every second arrangement were interchanged. For the next permutation the pictures in every pair of the first sequence were swapped. The third and fourth permutations are the reversed versions of the first and the second order. The results were saved in a database and collected for a duration of several weeks.

3.3 Participants

The experiment started on December the 30th. During the time-frame of about four weeks 267 persons attended the test. 60 of them provided suitable information in their online sessions. 16 of them did not fill out all or some of the personal data questions but completed the picture tests. Most of the participants (34) were students of different sciences. The subjects consisted of 25 female and 33 male persons, two did not give particulars. The age range was from 16 to 46 years, 25.0 on average.

3.4 Results

For the data analysis we used a sample of 60 complete data sets, i.e. only data sets without missing values in the two paired comparisons were used for testing our hypotheses. In the following, we present some descriptive information at first and then continue by showing the results of testing our hypotheses.

3.4.4 Descriptive analysis

We started analyzing our data by using descriptive statistics in order to get an overview of our results. Figure 7 and figure 8 show the means of the absolute frequencies of all ratios, i.e. how often each ratio has been preferred by all probands on average.

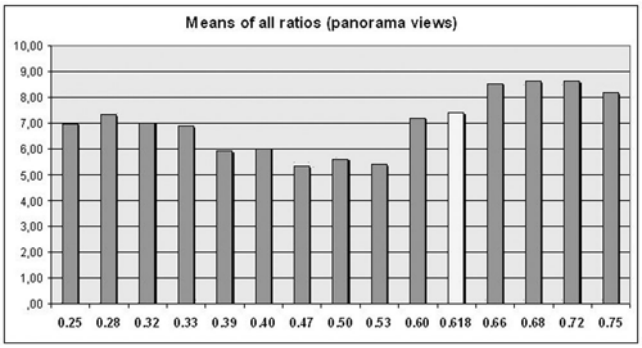

Figure 7: Means of all ratios (panorama views)

As can be seen on figure 7 the means of the absolute frequencies in the pair comparison of the panorama views can be clustered to three sections. On the left, there is an interval from 0.25 to 0.33 with moderate means. In the center, the interval from 0.39 to 0.53 has been least chosen and finally the interval from 0.60 to 0.75 has been most preferred.

The distribution of means is quite different for the pair comparison of rectangles. In figure 8 the means of the ratios which are on the left and right margin are the lowest. Whereas the ratios in the center have been preferred the most.

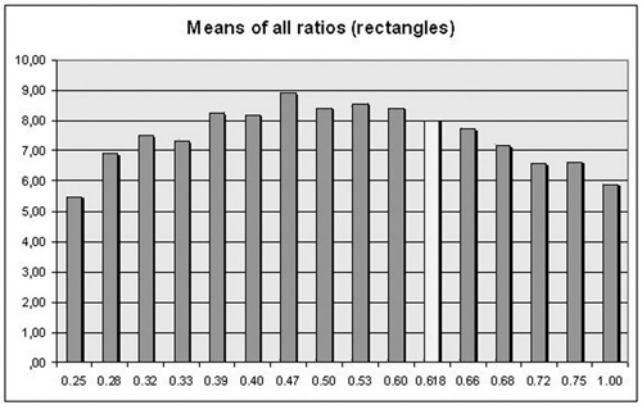

Figure 8: Means of all ratios (rectangles)

In order to have a closer look on the ratios 0.5 and 0.618 (Golden Section) we compared their means with all other ratios. Regarding the pair comparison of panorama views, the mean of 0.5 ($\bar{X}$ = 5:62) is smaller than the mean of the others ($\bar{X}$ = 7.1). In case of the Golden Section there was hardly a difference between the means (0.618: $\bar{X}$ = 7.4, all other ratios: $\bar{X}$ = 6.97). Repeating the same procedure for the pair comparison of rectangles, the mean of 0.5 ($\bar{X}$ = 8.4) was higher than the mean of all other ratios ($\bar{X}$ = 7.44) and the mean of 0.618 ($\bar{X}$ = 8.02) was a little higher than the mean of the rest ($\bar{X}$ = 7.47).

3.4.5 Testing of hypotheses

Before we investigated the main issue of our experiment, the question if there is a preference for the irrational, we tested if there are impacts of sequence and of picture type on the selection behavior. Independent samples t-Tests (level of significance: 0.05) were applied for these hypotheses.

Originally, we created four different sequences for presenting the two pair comparisons. Unfortunately, only two of the four sequences could be used due to technical problems. After testing the remaining two sequences, the result showed that there is no significant impact. We used four types of pictures which can be divided into abstract versus natural and into round versus oblong. Considering the condition abstract versus natural there was a only a significant influence on the ratio 0.6 (t = 2.386, df = 58) in the pair comparison of the panorama views and on the ratios 0.33 (t = -2.152, df = 58) and 0.53 (t = -2.026, df = 58) in the pair comparison of the rectangles. Thus, a systematical influence wasn't found. In case of the condition round versus oblong the results showed that there is a significant impact on the ratios 0.4 (t = 2.977, df = 58), 0.47 (t = 2.535, df = 58), 0.5 (t = 2.917, df = 58), 0.68 (t = -2.748, df = 58) and 0.75 (t = -2.308, df = 58) for the panorama views. On closer examination, the means of 0.4, 0.47 and 0.5 are significantly higher under the condition round and the means of 0.68 and 0.75 under the condition oblong.

The main issue of our experiment was the question if there is a preference for the irrational. Following this hypothesis we applied a one-sample t-test (level of significance: 0.05) on the data of the two pair comparisons (panorama views and rectangles) in order to find out if the irrational ratios are preferred. That means that we had to observe all pair comparisons between a rational and an irrational ratio and count how often the irrational ratio was preferred. Subsequently, we could test if the mean of all irrational ratios is higher than an expected value (the expected average assuming rectangular distribution). The result showed that the t-test for the panorama views is not significant, but it is in the case of the rectangles (t = 3.249, df = 59).

Additionally to our main hypothesis we also tested if there is a preference for the interval from 0.66 to 0.72. As it can be seen in figure 7 (descriptive analysis) this section has the highest values in the pair comparison of panorama views. An one-sample t-test (level of significance: 0.05) was significant for this interval (t = -3.565, df = 59).

Another question which we wanted to explore is the influence of personality, more precisely the impact of the need of security of a person. Here, independent samples t-Tests (level of significance: 0.05) were applied. Neither in the pair comparison of panorama views nor in the case of the rectangles was a significant influence of the personality on selecting the ratio 0.5, the most symmetrical configuration. However, there were significant influences on the ratios 0.68 ($t = -2.521$, $df = 58$) and 0.75 ($t = -1.942$, $df = 58$) for the panorama views. That means that probands who are classified ”secure” prefer these two ratios significantly. In case of the pair comparison of rectangles, there is an impact on the ratios 0.33 ($t = 2.097$, $df = 58$), 0.72 ($t = -2.068$, $df = 58$), 0.75 ($t = -2.078$, $df = 58$) and 1.00 ($t = -2.453$, $df = 58$). We also tested if there is an impact of the need for security on preferring the interval from 0.66 to 0.72. Here, we could find a significant influence. People which we classified as rather secure do prefer the interval more often than insecure probands ($t = 2.267$, $df = 58$).

Finally, we counted an correlation between the two pair comparisons. The Pearson correlation was -0.616 and significant at the 0.05 level. As we have already seen in the descriptive analysis the distributions of the means of the absolute frequencies are rather opposites.

4 Conclusion

We conducted an empirical study to supplement previous findings concerning the preference of the Golden Section. We argued that the Golden Section itself can neither be perceived nor displayed due to its high degree of irrationality. Therefore, previous results are open to alternative explanations: First, humans might prefer ratios in the area around the Golden Section but not necessarily the Golden Section itself. Second, humans might prefer ratios with a higher degree of irrationality over simple ratios and not the Golden Section specifically.

For the one-dimensional stimuli, ratios in the interval between 0.66 and 0.72 were preferred, regardless whether stimuli were natural or artificial, oblong or round. Surprisingly for the Fechner rectangles, that is in the two-dimensional case, the preferences were highly different with preferences near 0.5.

Since the experiment was conducted online, stimuli were presented on a computer monitor with a given breadth/width ratio. This might have an impact on preferences. Therefore, the experiment should be repeated in a paper and pencil variant.

5 References

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