Lecture 6: Inductive Logic Programming

Cognitive Systems - Machine Learning

Part II: Special Aspects of Concept Learning

FOIL, Inverted Resolution, Sequential Covering

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Motivation

- it is useful to learn the target function as a set of if-then-rules
  - one of the most expressive and human readable representations
  - e.g. decision trees
- **Inductive Logic Programming (ILP):**
  - rules are learned directly
  - designed to learn *first-order rules* (i.e. including variables)
  - *sequential covering* to incrementally grow the final set of rules
- PROLOG programs are sets of first-order rules

$\Rightarrow$ a general-purpose method capable of learning such rule sets may be viewed as an algorithm for automatically inferring PROLOG programs

Examples

- Propositional Logic:

```
IF Humidity=normal AND Outlook=sunny
THEN PlayTennis=yes
IF Humidity=normal AND Temperature=mild AND Wind=weak
THEN PlayTennis=yes
```

```
playTennis :- humidity(normal), outlook(sunny).
playTennis :- humidity(normal), temperature(mild),
              wind(weak).
```

- First Order Logic:

```
IF Parent(x,y) THEN Ancestor(x,y)
IF Parent(x,z) AND Ancestor(z,y) THEN Ancestor(x,y)
```

```
ancestor(X,Y) :- parent(X,Y).
ancestor(X,Y) :- parent(X,Z), ancestor(Z,Y).
```

Sequential Covering

- **basic strategy**: learn one rule, remove the data it covers, then iterate this process
- one of the most widespread approaches to learn a disjunctive set of rules (each rule itself is conjunctive)
- subroutine **LEARN-ONE-RULE**
  - accepts a set of positive and negative examples as input and outputs a single rule that covers many of the positive and few of the negative examples
  - **high accuracy**: predictions should be correct
  - **low coverage**: not neccessarily predictions for each example
- performs a greedy search without backtracking

  $\Rightarrow$ no guarantee to find the smallest or best set of rules

Sequential Covering

Algorithm

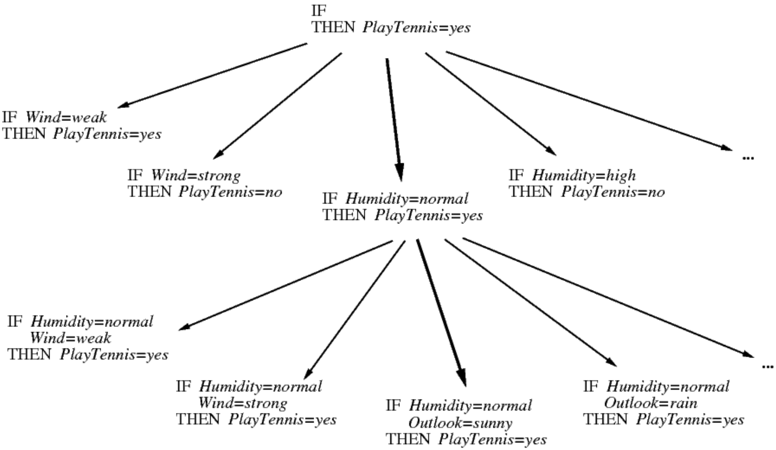
SEQUENTIAL-COVERING(*Target_attribute*, *Attributes*, *Examples*, *Threshold*)

- *Learned_Rules* $\leftarrow \{\}$
- *Rule* $\leftarrow$
  LEARN-ONE-RULE(*Target_attribute*, *Attributes*, *Examples*)
- While **PERFORMANCE**(*Rule*, *Examples*) $>$ *Threshold*, Do
  - *Learned_rules* $\leftarrow$ *Learned_rules* $+$ *Rule*
  - *Examples* $\leftarrow$ *Examples* $-$ { examples correctly classified by *Rule*}
  - *Rule* $\leftarrow$ **LEARN-ONE-RULE**(*Target_attribute*, *Attributes*, *Examples*)
- *Learned_rules* $\leftarrow$
  sort *Learned_rules* accord to **PERFORMANCE** over *Examples*
- return *Learned_rules*

General to Specific Beam Search

- **question**: How shall LEARN-ONE-RULE be designed to meet the needs of the sequential covering algorithm?
- organize the search through *H* analoguous to ID3
  - **but** follow only the most promising branch in the tree at each step
  - begin by considering the most general rule precondition (i.e. empty test)
  - then greedily add the attribute test that most improves rule performance over the training examples
  - unlike to ID3, this implementation follows only a single descendant at each search step rather than growing a subtree that covers all possible values of the selected attribute

General to Specific Beam Search



General to Specific Beam Search

- **so far** a local greedy search (analoguous to hill-climbing) is employed
  - danger of suboptimal results
  - susceptible to the typical hill-climbing problems

$\Rightarrow$ extension to **beam search**

  - $\Rightarrow$ algorithm maintains a list of the k best candidates at each step
  - $\Rightarrow$ at each step, descendants are generated for each of the k candidates and the resulting set is again reduced to the k most promising candidates

LEARN-ONE-RULE

LEARN-ONE-RULE(*Target_attribute*, *Attributes*, *Examples*, *k*)

Returns a single rule that covers some of the Examples. Conducts a general to specific greedy beam search for the best rule, guided by the PERFORMANCE metric.

- Initialize *Best_hypothesis* to the most general hyothesis $\emptyset$
- Initialize *Candidate_hypotheses* to the set {*Best_hypothesis*}
- While *Candidate_hypotheses* is not empty, Do
  1. *Generate the next more specific candidate_hypotheses*
     - *New_Candidate_hypotheses* $\leftarrow$ new generated and specialized candidates
  2. *Update Best_hypotheses*
     - Select hypothesis *h* from *New_candidate_hypotheses* with best PERFORMANCE over Examples
     - IF PERFORMANCE of *h* $>$ PERFORMANCE of *Best_hypothesis* THEN set *h* as new *Best_hypothesis*
  3. *Update Candidate_hypotheses*
     - *Candidate_hypotheses* $\leftarrow$ the *k* best members of *New_Candidate_hypotheses*

LEARN-ONE-RULE cont.

- Return a rule of the form

  "IF *Best_hypothesis* THEN *prediction*"
  where *prediction* is the most frequent value of *Target_attribute* among those *Examples* that match *Best_hypothesis*.

Example: Learn One Rule

- Learn *one* rule that covers a certain amount of positive examples
- with high accuracy
- remove the covered positive examples

Example

(s1)	Sky = sunny	(s2)	Sky = rainy
(a1)	AirTemp = warm	(a2)	AirTemp = cold
(h1)	Humidity = normal	(h2)	Humidity = high
(w1)	Water = warm	(w2)	Water = cool

Example cont.

- Assume $k = 4$
- Current most specific hypotheses: s1, s2, a1, a2, h1, h2, w1, w2
- Assume best possible hypotheses wrt performance P: s1, a2, h1, w1
- Generate new candidate hypotheses, e.g. by specializing s1:

  s1 & s1 (dublicate)

  s1 & s2 (inconsistent)

  s1 & a1

  s1 & a2

  s1 & h1

  ...

Performance Measures

- Relative Frequency (numbers of correctly classified examples by all examples)

$$\frac{n_c}{n}$$

- Entropy (S as set of examples that match precondition, p_i proportion of examples from S for which the target function takes the i-th value

$$-Entropy(S) = \sum_{i=1}^{c} p_i log_2 p_i$$

Sequential vs. Simultaneous Covering

- **sequential covering:**
  - learn just one rule at a time, remove the covered examples and repeat the process on the remaining examples
  - many search steps, making independent decisions to select each precondition for each rule
- **simultaneous covering:**
  - ID3 learns the entire set of disjunctive rules simultaneously as part of a single search for a decision tree
  - fewer search steps, because each choice influences the preconditions of all rules

$\Rightarrow$ Choice depends of how much data is available

  - plentiful $\rightarrow$ sequential covering (more steps supported)
  - scarce $\rightarrow$ simultaneous covering (decision sharing effective)

Differences in Search

- **generate-then-test:**
  - search through all syntactically legal hypotheses
  - generation of the successor hypotheses is only based on the syntax of the hypothesis representation
  - training data is considered after generation to choose among the candidate hypotheses
  - each training example is considered several times

  $\Rightarrow$ impact of noisy data is minimized

- **example driven:**
  - individual training examples constrain the generation of hypotheses
  - e.g. FIND-S, CANDIDATE ELIMINATION

  $\Rightarrow$ search can easily be misled

Learning First-Order Rules

- propositional expressions do not contain variables and are therefore less expressive than first-order expressions
- no general way to describe essential relations among the values of attributes
- ***Literals:*** We refer to atomic formulas also as atoms. Positive and negative atoms ($P, \neg P$) are called positive/negative literals
  e.g. $parent(x, y)$ or $\neg parent(x, y) \ldots$
- ***Clauses:*** A clause is a *disjunction* of positive and negative literals.
  e.g. $mother\_of(x, y) \vee father\_of(z, y)$

Learning First-Order Rules

Now we consider learning first-order rules (Horn Theories)

- a Horn clause is a clause containing at most *one positive literal*
- expression of the form:

$$H \vee \neg L_1 \vee \neg L_2 \vee ... \vee \neg L_n$$
$$\Longleftrightarrow H \leftarrow (L_1 \wedge L_2 \wedge ... \wedge L_n)$$
$$\Longleftrightarrow \text{IF } (L_1 \wedge L_2 \wedge ... \wedge L_n) \text{ THEN } H$$

- FOL terminology see *Intelligente Agenten*

FOIL (Quinlan, 1990)

- natural extension of SEQUENTIAL-COVERING and LEARN-ONE-RULE
- outputs sets of first-order rules similar to Horn Clauses with two exceptions
  1. **more restriced**, because literals are not permitted to contain function symbols
  2. **more expressive**, because literals in the body can be negated
- differences between FOIL and earlier algorithms:
  - seeks only rules that predict when the target literal is *True*
  - conducts a simple hill-climbing search instead of beam seach

FOIL

Algorithm

FOIL(*Target_predicate*, *Predicates*, *Examples*)

- *Pos* ← those *Examples* for which the *Target_predicate* is *True*
- *Neg* ← those *Examples* for which the *Target_predicate* is *False*
- *Learned_rules* ← {}
- while *Pos*, Do
  - *NewRule* ← the rule that predicts *Target_predicate* with no precondition
  - *NewRuleNeg* ← *Neg*
  - while *NewRuleNeg*, Do
    - *Candidate_literals* ← generate new literals for *NewRule*, based on *Predicates*
    - *Best_literal* ← $\max_{L \in Candidate\_literals}$ *FoilGain*(*L*, *NewRule*)
    - add *Best_literal* to preconditions of *NewRule*
    - *NewRuleNeg* ← subset of *NewRuleNeg* that satisfies *NewRule* preconditions
  - *Learned_rules* ← *Learned_rules* + *NewRule*
  - *Pos* ← *Pos* − { members of *Pos* covered by *NewRule*}
- Return *Learned_rules*

FOIL Hypothesis Space

- **outer loop (set of rules):**
  - specific-to-general search
  - initially, there are no rules, so that each example will be classified negative (most specific)
  - each new rule raises the number of examples classified as positive (more general)
  - disjunctive connection of rules
- **inner loop (preconditions for one rule):**
  - general-to-specific search
  - initially, there are no preconditions, so that each example satisfies the rule (most general)
  - each new precondition raises the number of examples classified as negative (more specific)
  - conjunctive connection of preconditions

Generating Candidate Specializations

- current rule:

  $P(x_1, x_2, ..., x_k) \leftarrow L_1 ... L_n$ where

  $L_1 ... L_n$ are the preconditions and
  $P(x_1, x_2, ..., x_k)$ is the head of the rule

- FOIL generates candidate specializations by considering new literals L_{n+1} that fit one of the following forms:
  - $Q(v_1, ..., v_r)$ where $Q \in \mathit{Predicates}$ and the v_i are new or already present variables (at least one v_i must already be present)
  - $\mathit{Equal}(x_j, x_k)$ where x_j and x_k are already present in the rule
  - the negation of either of the above forms

Training Data Example

Examples	Background Knowledge
GrandDaughter(Victor, Sharon)	Female(Sharon)
$\neg$ GrandDaugther(Tom, Bob)	Father(Sharon, Bob)
$\neg$ GrandDaughter(Victor, Victor)	Father(Tom, Bob)
	Father(Bob, Victor)

FOIL Example

Example

GrandDaughter(x,y) $\leftarrow$

Cadidate additions to precondition:
Equal(x,y), Female(x), Female(y), Father(x,y), Father(y,x), Father(x,z), Father(z,x), Father(z,y), and the negations to these literals

Assume greedy selection of Father(y,z):
GrandDaughter(x,y) $\leftarrow$ **Father(y,z)**

Candidate additions:
the ones from above and Female(z), Equal(z,y), Father(z,w), Father(w,z), and their negations

Learned Rule:
GrandDaughter(x,y) $\leftarrow$ **Father(y,z)** $\wedge$ **Father(z,x)** $\wedge$ **Female(y)**

FOIL Gain

$$FoilGain(L, R) \equiv t \left(log_2 \frac{p_1}{p_1 + n_1} - log_2 \frac{p_0}{p_0 + n_0} \right)$$

with

- L as new literal introduced in rule R to gain new rule R'
- t as number of positive bindings of rule R which are still covered by R'
- p_1 as number of positive bindings of rule R' and n_1 as number of negative bindings
- p_0 as number of positive bindings of rule R and n_0 as number of negative bindings

Remark: Bindings are the number of instantiations of the variables by constants. A binding is positive if the instantiated rule covers a positive example.

Learning Recursive Rule Sets

- Extend FOIL such that the target predicate can also be included in the preconditions with the same restrictions to variables as before.
- Problem: rule sets that produce infinite recurions
- FOIL uses a generate-and-test strategy alternatively recursive rule sets can be learned by analytical methods
  (see lecture inductive programming)

```
ancestor(X,Y) :- parent(X,Y).
ancestor(X,Y) :- parent(X,Z), ancestor(Z,Y).
```

Induction as Inverted Deduction

- **observation:** induction is just the inverse of deduction
- in general, machine learning involves building theories that explain the observed data
- Given some data D and some background knowledge B, learning can be described as generating a hypothesis h that, together with B, explains D.

$$(\forall < x_i, f(x_i) > \in D)(B \wedge h \wedge x_i) \vdash f(x_i)$$

- the above equation casts the learning problem in the framework of deductive inference and formal logic

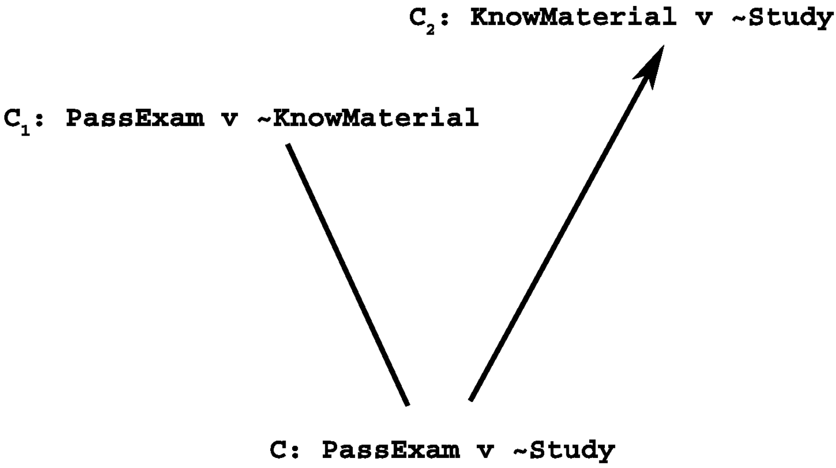
Induction as Inverted Deduction

- **features of inverted deduction:**
  - subsumes the common definition of learning as finding some general concept
  - background knowledge allows a more rich definition of when a hypothesis h is said to “fit” the data
- **practical difficulties:**
  - noisy data makes the logical framework completely lose the ability to distinguish between truth and falsehood
  - search is intractable
  - background knowledge often increases the complexity of H

Inverting Resolution

- **resolution** is a general method for automated deduction
- complete and sound method for deductive inference
- see CogSys1
- **Inverse Resolution Operator (propositional form):**
  1. Given initial clause C_1 and C, find a literal L that occurs in C_1 but not in clause C.
  2. Form the second clause C_2 by including the following literals
     $$C_2 = (C - (C_1 - \{L\})) \cup \{\neg L\}$$
- inverse resolution is not deterministic

Inverting Resolution



Inverting Resolution

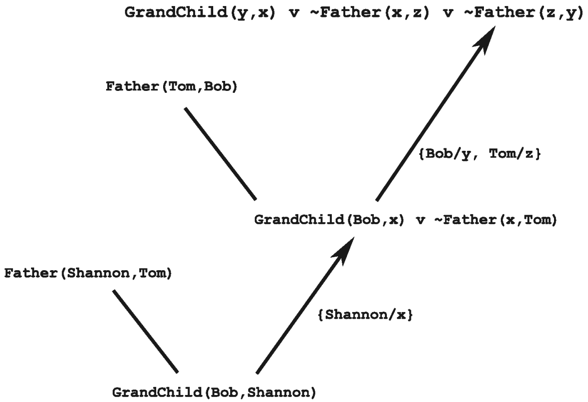
- **Inverse Resolution Operator (first-order form):**
  - resolution rule:
    1. Find a literal L_1 from clause C_1, literal L_2 from clause C_2, and substitution θ such that $L_1\theta = \neg L_2\theta$
    2. Form the resolvent C by including all literals from $C_1\theta$ and $C_2\theta$, except for $L_1\theta$ and $\neg L_2\theta$. That is,
       $C = (C_1 - \{L_1\})\theta \cup (C_2 - \{L_2\})\theta$
  - analytical derivation of the inverse resolution rule:

$$C = (C_1 - \{L_1\})\theta_1 \cup (C_2 - \{L_2\})\theta_2 \text{ where } \theta = \theta_1\theta_2$$

$$C - (C_1 - \{L_1\})\theta_1 = (C_2 - \{L_2\})\theta_2 \text{ where } L_2 = \neg L_1\theta_1\theta_2^{-1}$$

$$\Rightarrow C_2 = (C - (C_1 - \{L_1\})\theta_1)\theta_2^{-1} \cup \{\neg L_1\theta_1\theta_2^{-1}\}$$

Inverting Resolution



$D = \{GrandChild(Bob, Shannon)\}$
$B = \{Father(Shannon, Tom), Father(Tom, Bob)\}$

Remarks

- Inverse resolution leads to combinatorial explosion of candidate hypotheses
  - many possibilities to combine hypotheses with background-knowledge in order to generate more specific hyotheses
- Other techniques:
  - θ**-Subsumption** (used by *GOLEM*)
    replace terms by variables (interverted unification)
  - **inverse entailment** (used by *PROGOL*)
    generates just a single more specific hypothesis that entails the observed data

Generalization, θ-Subsumption, Entailment

- interesting to consider the relationship between the *more_general_than* relation and inverse entailment
- *more_general_than*: $h_j \geq_g h_k$ iff $(\forall x \in X)[h_k(x) \rightarrow h_j(x)]$. A hypothesis can also be expressed as $c(x) \leftarrow h(x)$.
- $\theta -$ *subsumption*: Consider two clauses C_j and C_k, both of the form $H \vee L_1 \vee ... \vee L_n$, where H is a positive literal and the L_i are arbitrary literals. Clause C_j is said to $\theta -$ *subsume* clause C_k iff $(\exists\theta)[C_j\theta \subseteq C_k]$.
- *Entailment*: Consider two clauses C_j and C_k. Clause C_j is said to *entail* clause C_k (written $C_j \vdash C_k$) iff C_k follows deductively from C_j.

Generalization, θ-Subsumption, Entailment

- if $h_1 \geq_g h_2$ then $C_1 : c(x) \leftarrow h_1(x)$ θ-subsumes $C_2 : c(x) \leftarrow h_2(x)$
- furthermore, θ-subsumption can hold even when the clauses have different heads

  $A : Mother(x, y) \leftarrow Father(x, z) \wedge Spouse(z, y)$
  $B : Mother(x, L) \leftarrow Father(x, B) \wedge Spouse(B, y) \wedge Female(x)$

  $A\theta \subseteq B$ if we choose $\theta = \{y/L, z/B\}$
- θ-subsumption is a special case of entailment

  $A : Elephant(father\_of(x)) \leftarrow Elephant(x)$
  $B : Elephant(father\_of(father\_of(y))) \leftarrow Elephant(y)$

  $A \vdash B$, but $\neg\exists\theta[A\theta \subseteq B]$

Generalization, θ-Subsumption, Entailment

- Generalization is a special case of θ-Subsumption
- θ-Subsumption is a special case of entailment
- In its most general form, inverse entailment produces intractable searches
- θ-Subsumption provides a convenient notion that lies midway between generalization and entailment!

Summary

- learns sets of first-order rules directly
- sequential covering algorithms learn just one rule at a time and perform many search steps
- hence, applicable if data is plentiful
- **FOIL** is a sequential covering algorithm
- a specific-to-general search is performed to form the result set
- a general-to-specific search is performed to form each new rule
- Induction can be viewed as the inverse of deduction
- hence, an inverse resolution operator can be found

Summary cont.

- Hypothesis language of Horn Clauses is more expressive than feature vectors (allowing variables, representing relations)
- Suitable for structured data (meshes, chemical structures, graph-representations in general)
- Not only for learning classifiers but also for learning general (recursive) programs (inductive programming)

Learning Terminology

FOIL

Supervised Learning	unsupervised learning

Approaches:

Concept / Classification	Policy Learning
symbolic	statistical / neuronal network
inductive	analytical

Learning Strategy:
$\Rightarrow$ **learning from examples**