Lecture 6: Inductive Logic Programming
Cognitive Systems - Machine Learning

Part II: Special Aspects of Concept Learning

FOIL, Inverted Resolution, Sequential Covering

last change November 24, 2011
Motivation

- it is useful to learn the target function as a set of if-then-rules
  - one of the most expressive and human readable representations
  - e.g. decision trees

- Inductive Logic Programming (ILP):
  - rules are learned directly
  - designed to learn first-order rules (i.e. including variables)
  - sequential covering to incrementally grow the final set of rules

- PROLOG programs are sets of first-order rules

⇒ a general-purpose method capable of learning such rule sets may be viewed as an algorithm for automatically inferring PROLOG programs
Examples

- **Propositional Logic:**

  IF Humidity=normal AND Outlook=sunny
  THEN PlayTennis=yes
  IF Humidity=normal AND Temperature=mild AND Wind=weak
  THEN PlayTennis=yes

  playTennis :- humidity(normal), outlook(sunny).
  playTennis :- humidity(normal), temperature(mild), wind(weak).

- **First Order Logic:**

  IF Parent(x,y) THEN Ancestor(x,y)
  IF Parent(x,z) AND Ancestor(z,y) THEN Ancestor(x,y)

  ancestor(X,Y) :- parent(X,Y).
  ancestor(X,Y) :- parent(X,Z), ancestor(Z,Y).
Sequential Covering

- **basic strategy**: learn one rule, remove the data it covers, then iterate this process

- one of the most widespread approaches to learn a disjunctive set of rules (each rule itself is conjunctive)

- subroutine **LEARN-ONE-RULE**
  - accepts a set of positive and negative examples as input and outputs a single rule that covers many of the positive and few of the negative examples
  - **high accuracy**: predictions should be correct
  - **low coverage**: not necessarily predictions for each example

- performs a greedy search without backtracking
  - no guarantee to find the smallest or best set of rules
Sequential Covering

**Algorithm**

**SEQUENTIAL-COVERING** *(Target_attribute, Attributes, Examples, Threshold)*

- $\textbf{Learned}_\text{Rules} \leftarrow \{\}$
- $\textbf{Rule} \leftarrow \text{LEARN-ONE-RULE}(\text{Target}_\text{attribute}, \text{Attributes}, \text{Examples})*

While $\text{PERFORMANCE}(\textbf{Rule}, \text{Examples}) > \text{Threshold}$, Do

- $\textbf{Learned}_\text{rules} \leftarrow \textbf{Learned}_\text{rules} + \textbf{Rule}$
- $\text{Examples} \leftarrow \text{Examples} - \{\text{ examples correctly classified by } \textbf{Rule}\}$
- $\textbf{Rule} \leftarrow \text{LEARN-ONE-RULE}(\text{Target}_\text{attribute}, \text{Attributes}, \text{Examples})*$

- $\textbf{Learned}_\text{rules} \leftarrow$
- sort $\textbf{Learned}_\text{rules}$ accord to $\text{PERFORMANCE}$ over $\text{Examples}$

- return $\textbf{Learned}_\text{rules}$
General to Specific Beam Search

**question:** How shall LEARN-ONE-RULE be designed to meet the needs of the sequential covering algorithm?

organize the search through $H$ analogous to ID3

- **but** follow only the most promising branch in the tree at each step
- begin by considering the most general rule precondition (i.e. empty test)
- then greedily add the attribute test that most improves rule performance over the training examples
- unlike to ID3, this implementation follows only a single descendant at each search step rather than growing a sub-tree that covers all possible values of the selected attribute
General to Specific Beam Search

IF Wind=weak
   THEN PlayTennis=yes

IF Wind=strong
   THEN PlayTennis=no

IF Humidity=high
   THEN PlayTennis=no

IF Humidity=normal
   THEN PlayTennis=yes

IF Humidity=normal
   Wind=weak
   THEN PlayTennis=yes

IF Humidity=normal
   Wind=strong
   THEN PlayTennis=yes

IF Humidity=normal
   Outlook=sunny
   THEN PlayTennis=yes

IF Humidity=normal
   Outlook=rain
   THEN PlayTennis=Yes
General to Specific Beam Search

- so far a local greedy search (analogous to hill-climbing) is employed
  - danger of suboptimal results
  - susceptible to the typical hill-climbing problems

⇒ extension to beam search

⇒ algorithm maintains a list of the $k$ best candidates at each step
⇒ at each step, descendants are generated for each of the $k$ candidates and the resulting set is again reduced to the $k$ most promising candidates
LEARN-ONE-RULE

**LEARN-ONE-RULE**(*Target_attribute, Attributes, Examples, k*)

Returns a single rule that covers some of the Examples. Conducts a general to specific greedy beam search for the best rule, guided by the PERFORMANCE metric.

- Initialize *Best_hypothesis* to the most general hypothesis \(\emptyset\)
- Initialize *Candidate_hypotheses* to the set \{*Best_hypothesis*\}
- While *Candidate_hypotheses* is not empty, Do
  1. **Generate the next more specific candidate_hypotheses**
     - \(\text{New}_\text{Candidate}_\text{hypotheses} \leftarrow\) new generated and specialized candidates
  2. **Update Best_hypotheses**
     - Select hypothesis \(h\) from \(\text{New}_\text{candidate}_\text{hypotheses}\) with best PERFORMANCE over Examples
     - IF PERFORMANCE of \(h >\) PERFORMANCE of *Best_hypothesis* THEN set \(h\) as new *Best_hypothesis*
  3. **Update Candidate_hypotheses**
     - \(\text{Candidate}_\text{hypotheses} \leftarrow\) the \(k\) best members of \(\text{New}_\text{Candidate}_\text{hypotheses}\)
Return a rule of the form

“IF Best_hypothesis THEN prediction”

where prediction is the most frequent value of Target_attribute among those Examples that match Best_hypothesis.
Example: Learn One Rule

- Learn one rule that covers a certain amount of positive examples with high accuracy.
- Remove the covered positive examples.

Example

<table>
<thead>
<tr>
<th>(s1)</th>
<th>Sky = sunny</th>
<th>(s2)</th>
<th>Sky = rainy</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a1)</td>
<td>AirTemp = warm</td>
<td>(a2)</td>
<td>AirTemp = cold</td>
</tr>
<tr>
<td>(h1)</td>
<td>Humidity = normal</td>
<td>(h2)</td>
<td>Humidity = high</td>
</tr>
<tr>
<td>(w1)</td>
<td>Water = warm</td>
<td>(w2)</td>
<td>Water = cool</td>
</tr>
</tbody>
</table>
Assume $k = 4$

Current most specific hypotheses: $s_1, s_2, a_1, a_2, h_1, h_2, w_1, w_2$

Assume best possible hypotheses wrt performance $P$: $s_1, a_2, h_1, w_1$

Generate new candidate hypotheses, e.g. by specializing $s_1$:

- $s_1 \& s_1$ (duplicate)
- $s_1 \& s_2$ (inconsistent)
- $s_1 \& a_1$
- $s_1 \& a_2$
- $s_1 \& h_1$
...


Performance Measures

- Relative Frequency (numbers of correctly classified examples by all examples)
  \[ \frac{n_c}{n} \]

- Entropy (\( S \) as set of examples that match precondition, \( p_i \) proportion of examples from \( S \) for which the target function takes the \( i \)-th value)
  \[ -\text{Entropy}(S) = \sum_{i=1}^{c} p_i \log_2 p_i \]
Sequential vs. Simultaneous Covering

- **sequential covering:**
  - learn just one rule at a time, remove the covered examples and repeat the process on the remaining examples
  - many search steps, making independent decisions to select each precondition for each rule

- **simultaneous covering:**
  - ID3 learns the entire set of disjunctive rules simultaneously as part of a single search for a decision tree
  - fewer search steps, because each choice influences the preconditions of all rules

⇒ Choice depends of how much data is available
  - plentiful → sequential covering (more steps supported)
  - scarce → simultaneous covering (decision sharing effective)
Differences in Search

**generate-then-test:**
- search through all syntactically legal hypotheses
- generation of the successor hypotheses is only based on the syntax of the hypothesis representation
- training data is considered after generation to choose among the candidate hypotheses
- each training example is considered several times
  - impact of noisy data is minimized

**example driven:**
- individual training examples constrain the generation of hypotheses
- e.g. FIND-S, CANDIDATE ELIMINATION
  - search can easily be misled
Learning First-Order Rules

- propositional expressions do not contain variables and are therefore less expressive than first-order expressions

- no general way to describe essential relations among the values of attributes

- **Literals:** We refer to atomic formulas also as atoms. Positive and negative atoms ($P, \neg P$) are called positive/negative literals.
  - e.g. $parent(x,y)$ or $\neg parent(x,y)$...

- **Clauses:** A clause is a disjunction of positive and negative literals.
  - e.g. $mother\_of(x,y) \lor father\_of(z,y)$
Learning First-Order Rules

Now we consider learning first-order rules (Horn Theories)

- a **Horn clause** is a clause containing at most *one positive literal*

- expression of the form:
  
  \[ H \lor \neg L_1 \lor \neg L_2 \lor \ldots \lor \neg L_n \]

  \[ \iff H \iff (L_1 \land L_2 \land \ldots \land L_n) \]

  \[ \iff \text{IF} (L_1 \land L_2 \land \ldots \land L_n) \text{ THEN } H \]

- FOL terminology see *Intelligente Agenten*
FOIL (Quinlan, 1990)

- natural extension of SEQUENTIAL-COVERING and LEARN-ONE-RULE
- outputs sets of first-order rules similar to Horn Clauses with two exceptions
  1. more restricted, because literals are not permitted to contain function symbols
  2. more expressive, because literals in the body can be negated
- differences between FOIL and earlier algorithms:
  - seeks only rules that predict when the target literal is True
  - conducts a simple hill-climbing search instead of beam search
Algorithm

**FOIL**(*Target\_predicate*, *Predicates*, *Examples*)

1. $Pos \leftarrow$ those *Examples* for which the *Target\_predicate* is True
2. $Neg \leftarrow$ those *Examples* for which the *Target\_predicate* is False
3. $Learned\_rules \leftarrow \{\}$
4. while $Pos$, Do
   1. $NewRule \leftarrow$ the rule that predicts *Target\_predicate* with no precondition
   2. $NewRuleNeg \leftarrow$ $Neg$
   3. while $NewRuleNeg$, Do
      1. $Candidate\_literals \leftarrow$ generate new literals for $NewRule$, based on *Predicates*
      2. $Best\_literal \leftarrow \max_{L \in Candidate\_literals} FoilGain(L, NewRule)$
      3. add $Best\_literal$ to preconditions of $NewRule$
      4. $NewRuleNeg \leftarrow$ subset of $NewRuleNeg$ that satisfies $NewRule$ preconditions
      5. $Learned\_rules \leftarrow$ $Learned\_rules + NewRule$
      6. $Pos \leftarrow$ $Pos - \{ \text{members of } Pos \text{ covered by } NewRule\}$
   4. Return $Learned\_rules$
FOIL Hypothesis Space

- **outer loop (set of rules):**
  - specific-to-general search
  - initially, there are no rules, so that each example will be classified negative (most specific)
  - each new rule raises the number of examples classified as positive (more general)
  - disjunctive connection of rules

- **inner loop (preconditions for one rule):**
  - general-to-specific search
  - initially, there are no preconditions, so that each example satisfies the rule (most general)
  - each new precondition raises the number of examples classified as negative (more specific)
  - conjunctive connection of preconditions
Generating Candidate Specializations

- current rule:
  \[ P(x_1, x_2, \ldots, x_k) \leftarrow L_1 \ldots L_n \]\n  where
  \( L_1 \ldots L_n \) are the preconditions and
  \( P(x_1, x_2, \ldots, x_k) \) is the head of the rule

- FOIL generates candidate specializations by considering new literals \( L_{n+1} \) that fit one of the following forms:
  - \( Q(v_1, \ldots, v_r) \) where \( Q \in \text{Predicates} \) and the \( v_i \) are new or already present variables (at least one \( v_i \) must already be present)
  - \( Equal(x_j, x_k) \) where \( x_j \) and \( x_k \) are already present in the rule
  - the negation of either of the above forms
## Training Data Example

<table>
<thead>
<tr>
<th>Examples</th>
<th>Background Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>GrandDaughter(Victor, Sharon)</td>
<td>Female(Sharon)</td>
</tr>
<tr>
<td>¬ GrandDaughter(Tom, Bob)</td>
<td>Father(Sharon, Bob)</td>
</tr>
<tr>
<td>¬ GrandDaughter(Victor, Victor)</td>
<td>Father(Tom, Bob)</td>
</tr>
<tr>
<td></td>
<td>Father(Bob, Victor)</td>
</tr>
</tbody>
</table>
Example

\textbf{GrandDaughter}(x,y) \leftarrow

Candidate additions to precondition:
\textit{Equal}(x,y), \textit{Female}(x), \textit{Female}(y), \textit{Father}(x,y), \textit{Father}(y,x), \textit{Father}(x,z), \textit{Father}(z,x), \textit{Father}(z,y), \text{and the negations to these literals}

Assume greedy selection of \textit{Father}(y,z):
\textbf{GrandDaughter}(x,y) \leftarrow \textit{Father}(y,z)

Candidate additions:
\textit{the ones from above and Female}(z), \textit{Equal}(z,y), \textit{Father}(z,w), \textit{Father}(w,z), \text{and their negations}

Learned Rule:
\textbf{GrandDaughter}(x,y) \leftarrow \textit{Father}(y,z) \land \textit{Father}(z,x) \land \textit{Female}(y)
FOIL Gain

\[
FoilGain(L, R) \equiv t \left( \log_2 \frac{p_1}{p_1 + n_1} - \log_2 \frac{p_0}{p_0 + n_0} \right)
\]

with

- \( L \) as new literal introduced in rule \( R \) to gain new rule \( R' \)
- \( t \) as number of positive bindings of rule \( R \) which are still covered by \( R' \)
- \( p_1 \) as number of positive bindings of rule \( R' \) and \( n_1 \) as number of negative bindings
- \( p_0 \) as number of positive bindings of rule \( R \) and \( n_0 \) as number of negative bindings

Remark: Bindings are the number of instantiations of the variables by constants. A binding is positive if the instantiated rule covers a positive example.
Learning Recursive Rule Sets

- Extend FOIL such that the target predicate can also be included in the preconditions with the same restrictions to variables as before.

- Problem: rule sets that produce infinite recursions

- FOIL uses a generate-and-test strategy alternatively recursive rule sets can be learned by analytical methods (see lecture inductive programming)

\[
\text{ancestor}(X,Y) \leftarrow \text{parent}(X,Y). \\
\text{ancestor}(X,Y) \leftarrow \text{parent}(X,Z), \text{ancestor}(Z,Y).
\]
**observation:** induction is just the inverse of deduction

in general, machine learning involves building theories that explain the observed data

Given some data $D$ and some background knowledge $B$, learning can be described as generating a hypothesis $h$ that, together with $B$, explains $D$.

$$(\forall <x_i, f(x_i) \in D)(B \land h \land x_i) \vdash f(x_i)$$

the above equation casts the learning problem in the framework of deductive inference and formal logic
features of inverted deduction:
- subsumes the common definition of learning as finding some general concept
- background knowledge allows a more rich definition of when a hypothesis $h$ is said to “fit” the data

practical difficulties:
- noisy data makes the logical framework completely lose the ability to distinguish between truth and falsehood
- search is intractable
- background knowledge often increases the complexity of $H$
Inverting Resolution

- **resolution** is a general method for automated deduction
- complete and sound method for deductive inference
- see *Intelligente Agenten*

**Inverse Resolution Operator (propositional form):**

1. Given initial clause $C_1$ and $C$, find a literal $L$ that occurs in $C_1$ but not in clause $C$.
2. Form the second clause $C_2$ by including the following literals
   \[ C_2 = (C - (C_1 - \{L\})) \cup \{\neg L\} \]

- inverse resolution is not deterministic
Inverting Resolution

\[ C_1: \text{PassExam} \lor \neg\text{KnowMaterial} \]

\[ C_2: \text{KnowMaterial} \lor \neg\text{Study} \]

\[ C: \text{PassExam} \lor \neg\text{Study} \]
**Inverse Resolution Operator (first-order form):**

- **resolution rule:**
  1. Find a literal $L_1$ from clause $C_1$, literal $L_2$ from clause $C_2$, and substitution $\theta$ such that $L_1 \theta = \neg L_2 \theta$
  2. Form the resolvent $C$ by including all literals from $C_1 \theta$ and $C_2 \theta$, except for $L_1 \theta$ and $\neg L_2 \theta$. That is,
     \[
     C = (C_1 - \{L_1\}) \theta_1 \cup (C_2 - \{L_2\}) \theta_2
     \]

- **analytical derivation of the inverse resolution rule:**
  \[
  C = (C_1 - \{L_1\}) \theta_1 \cup (C_2 - \{L_2\}) \theta_2 \text{ where } \theta = \theta_1 \theta_2
  \]
  \[
  C - (C_1 - \{L_1\}) \theta_1 = (C_2 - \{L_2\}) \theta_2 \text{ where } L_2 = \neg L_1 \theta_1 \theta_2^{-1}
  \]
  \[
  \Rightarrow C_2 = (C - (C_1 - \{L_1\}) \theta_1) \theta_2^{-1} \cup \{\neg L_1 \theta_1 \theta_2^{-1}\}
  \]
Inverting Resolution

\[ D = \{ \text{GrandChild}(Bob, Shannon) \} \]
\[ B = \{ \text{Father}(Shannon, Tom), \text{Father}(Tom, Bob) \} \]
Remarks

- Inverse resolution leads to combinatorial explosion of candidate hypotheses
  - many possibilities to combine hypotheses with background-knowledge in order to generate more specific hypotheses

- Other techniques:
  - \( \theta \)-Subsumption (used by GOLEM)
    replace terms by variables (interverted unification)
  - inverse entailment (used by PROGOL)
    generates just a single more specific hypothesis that entails the observed data
Generalization, $\theta$-Subsumption, Entailment

- interesting to consider the relationship between the *more_general_than* relation and inverse entailment

more_general_than: $h_j \geq_g h_k$ iff $(\forall x \in X)[h_k(x) \rightarrow h_j(x)]$. A hypothesis can also be expressed as $c(x) \leftarrow h(x)$.

$\theta$ – *subsumption*: Consider two clauses $C_j$ and $C_k$, both of the form $H \lor L_1 \lor \ldots \lor L_n$, where $H$ is a positive literal and the $L_i$ are arbitrary literals. Clause $C_j$ is said to $\theta$ – *subsume* clause $C_k$ iff $(\exists \theta)[C_j \theta \subseteq C_k]$.

Entailment: Consider two clauses $C_j$ and $C_k$. Clause $C_j$ is said to *entail* clause $C_k$ (written $C_j \vdash C_k$) iff $C_k$ follows deductively from $C_j$. 
Generalization, $\theta$-Subsumption, Entailment

- if $h_1 \geq_g h_2$ then $C_1 : c(x) \leftarrow h_1(x) \theta$-subsumes $C_2 : c(x) \leftarrow h_2(x)$

- furthermore, $\theta$-subsumption can hold even when the clauses have different heads
  
  $A : \text{Mother}(x, y) \leftarrow \text{Father}(x, z) \land \text{Spouse}(z, y)$
  $B : \text{Mother}(x, L) \leftarrow \text{Father}(x, B) \land \text{Spouse}(B, y) \land \text{Female}(x)$

  $A \theta \subseteq B$ if we choose $\theta = \{y/L, z/B\}$

- $\theta$-subsumption is a special case of entailment
  
  $A : \text{Elephant}(\text{father\_of}(x)) \leftarrow \text{Elephant}(x)$
  $B : \text{Elephant}(\text{father\_of}(\text{father\_of}(y))) \leftarrow \text{Elephant}(y)$

  $A \vdash B$, but $\not\exists \theta[A \theta \subseteq B]$
Generalization, $\theta$-Subsumption, Entailment

- Generalization is a special case of $\theta$-Subsumption
- $\theta$-Subsumption is a special case of entailment
- In its most general form, inverse entailment produces intractable searches
- $\theta$-Subsumption provides a convenient notion that lies midway between generalization and entailment!
Summary

- learns sets of first-order rules directly
- sequential covering algorithms learn just one rule at a time and perform many search steps
- hence, applicable if data is plentiful

**FOIL** is a sequential covering algorithm
- a specific-to-general search is performed to form the result set
- a general-to-specific search is performed to form each new rule

Induction can be viewed as the inverse of deduction
- hence, an inverse resolution operator can be found
Summary cont.

- Hypothesis language of Horn Clauses is more expressive than feature vectors (allowing variables, representing relations)
- Suitable for structured data (meshes, chemical structures, graph-representations in general)
- Not only for learning classifiers but also for learning general (recursive) programs (inductive programming)
## Learning Terminology

### FOIL

<table>
<thead>
<tr>
<th>Supervised Learning</th>
<th>unsupervised learning</th>
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**Approaches:**

<table>
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<th>Concept / Classification</th>
<th>Policy Learning</th>
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<tbody>
<tr>
<td>symbolic</td>
<td>statistical / neuronal network</td>
</tr>
<tr>
<td>inductive</td>
<td>analytical</td>
</tr>
</tbody>
</table>

**Learning Strategy:**

⇒ learning from examples