Lecture 12: Reinforcement Learning

Cognitive Systems - Machine Learning

Part III: Learning Programs and Strategies

Q Learning, Dynamic Programming

last change February 23, 2015

Motivation

- **addressed problem**: How can an autonomous agent that senses and acts in its environment learn to choose optimal actions to achieve its goals?
- consider building a learning robot (i.e., agent)
  - the agent has a set of *sensors* to observe the *state* of its environment and
  - a set of *actions* it can perform to alter its state
  - the task is to learn a control strategy, or *policy*, for choosing actions that achieve its goals
- **assumption:** goals can be defined by a *reward function* that assigns a numerical value to each distinct action the agent may perform from each distinct state

Motivation

- **considered settings:**
  - deterministic or non-deterministic outcomes
  - prior background knowledge available or not
- **similarity to function approximation:**
  - approximating the function $\pi : S \rightarrow A$

    where S is the set of states and A the set of actions
- **differences to function approximation:**
  - Delayed reward: training information is not available in the form $< s, \pi(s) >$. Instead the trainer provides only a sequence of immediate reward values.
  - Temporal credit assignment: determining which actions in the sequence are to be credited with producing the eventual reward

Motivation

- **differences to function approximation (cont.):**
  - exploration: distribution of training examples is influenced by the chosen action sequence
    - which is the most effective exploration strategy?
    - trade-off between exploration of unknown states and exploitation of already known states
  - partially observable states: sensors only provide partial information of the current state (e.g. forward-pointing camera, dirty lenses)
  - life-long learning: function approximation often is an isolated task, while robot learning requires to learn several related tasks within the same environment

Outline

- The Learning Task
- *Q* Learning
  - Algorithm
- Experimentation Strategies
- Generalizing From Examples
- Relationship to Dynamic Programming
- Advanced Topics

The Learning Task

- based on Markov Decision Processes (MDP)
  - the agent can perceive a set S of distinct states of its environment and has a set A of actions that it can perform
  - at each discrete time step t, the agent senses the **current state** s_t, chooses a **current action** a_t and performs it
  - the environment responds by returning a **reward** $r_t = r(s_t, a_t)$ and by producing the **successor state** $s_{t+1} = \delta(s_t, a_t)$
  - the functions r and δ are part of the environment and not necessarily known to the agent
  - in an MDP, the functions $r(s_t, a_t)$ and $\delta(s_t, a_t)$ depend only on the current state and action

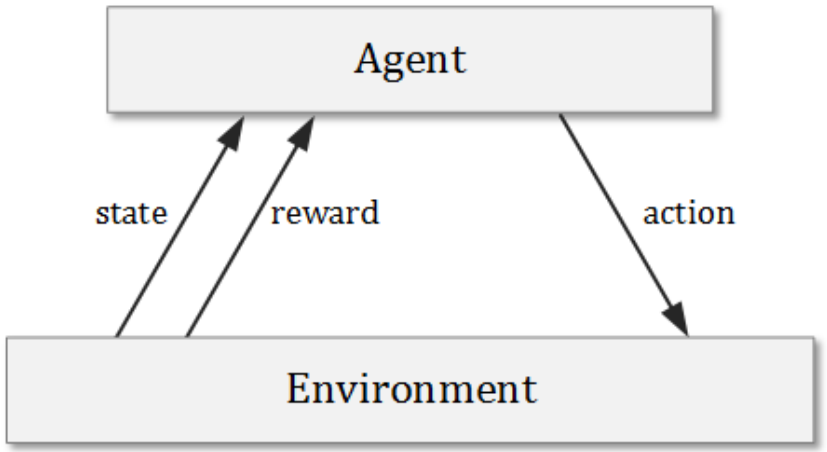
The Learning Task

- the task is to learn a policy $\pi : S \rightarrow A$
- one approach to specify which policy π the agent should learn is to require the policy that produces the greatest possible cumulative reward over time (**discounted cumulative reward**)

$$V^{\pi}(s_t) \equiv r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} \ \ldots$$
$$\equiv \sum_{i=0}^{\infty} \gamma^i r_{t+i}$$

where $V^{\pi}(s_t)$ is the cumulative value achieved by following an arbitrary policy π from an arbitrary initial state s_t
r_{t+i} is generated by repeatedly using the policy π and γ $(0 \leq \gamma < 1)$ is a constant that determines the relative value of delayed versus immediate rewards

The Learning Task

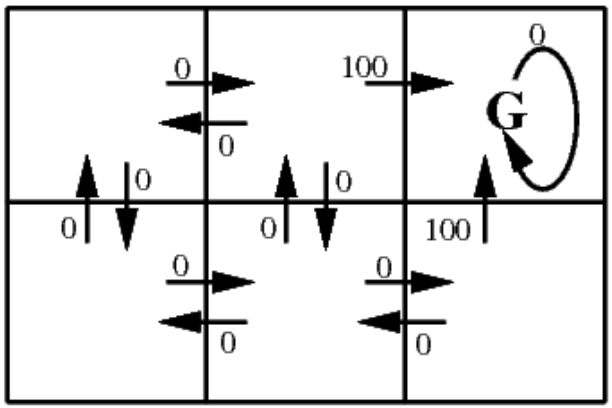


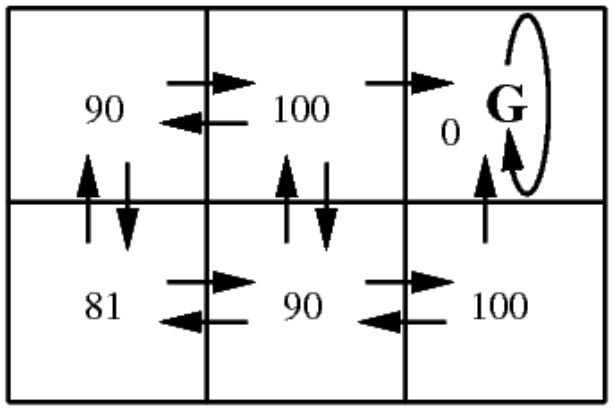
Goal: Learn to choose actions that maximize
$r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots$, where $0 \leq \gamma \leq 1$

- hence, the agent's learning task can be formulated as

$$\pi^* \equiv \underset{\pi}{argmax}\ V^{\pi}(s), (\forall s)$$

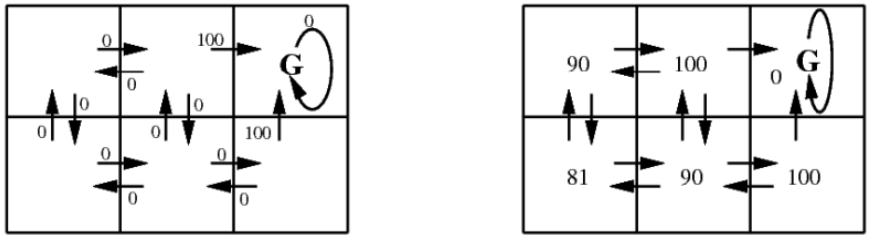
Illustrative Example





- the left diagram depicts a simple grid-world environment
  - squares $\approx$ states, locations
  - arrows $\approx$ possible transitions (with annotated $r(s, a)$)
  - $G \approx$ goal state (absorbing state)
- $\gamma = 0.9$
- once states, actions and rewards are defined and γ is chosen, the optimal policy π^* with its value function $V^*(s)$ can be determined

Illustrative Example



- the right diagram shows the values of V^* for each state
- e.g. consider the bottom-right state
  - $V^* = 100$, because π^* selects the "move up" action that receives a reward of 100
  - thereafter, the agent will stay in G and receive no further awards
  - $V^* = 100 + \gamma \cdot 0 + \gamma^2 \cdot 0 + ... = 100$
- e.g. consider the bottom-center state
  - $V^* = 90$, because π^* selects the "move right" and "move up" actions
  - $V^* = 0 + \gamma \cdot 100 + \gamma^2 \cdot 0 + ... = 90$
- recall that V^* is defined to be the sum of discounted future awards over **infinite** future

Q Learning

- it is easier to learn a numerical evaluation function than implement the optimal policy in terms of the evaluation function
- **question:** What evaluation function should the agent attempt to learn?
- one obvious choice is V^*
- the agent should prefer s_1 to s_2 whenever $V^*(s_1) > V^*(s_2)$
- **problem:** the agent has to chose among actions, not among states

$$\pi^*(s) = \underset{a}{argmax}[r(s,a) + \gamma V^*(\delta(s,a))]$$

the optimal action in state s is the action a that maximizes the sum of the immediate reward $r(s,a)$ plus the value of V^* of the immediate successor, discounted by γ

Q Learning

- thus, the agent can acquire the optimal policy by learning V^*, *provided it has perfect knowledge of the immediate reward function r* and the state transition function δ
- in many problems, it is impossible to predict in advance the exact outcome of applying an arbitrary action to an arbitrary state
- the Q function provides a solution to this problem
  - $Q(s, a)$ indicates the maximum discounted reward that can be achieved starting from s and applying action a first

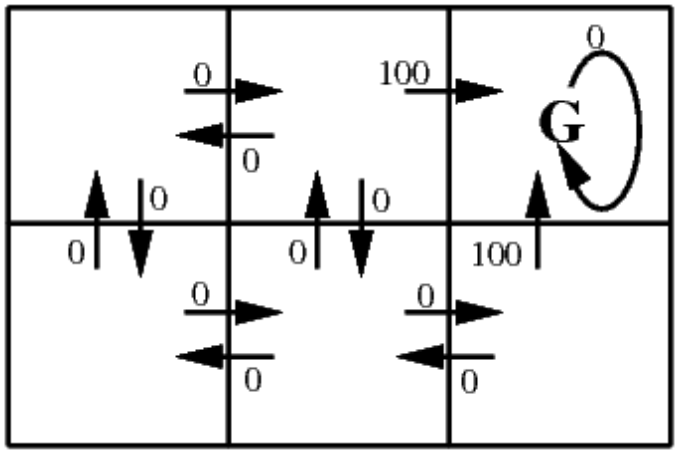
$$Q(s, a) = r(s, a) + \gamma V^*(\delta(s, a))$$

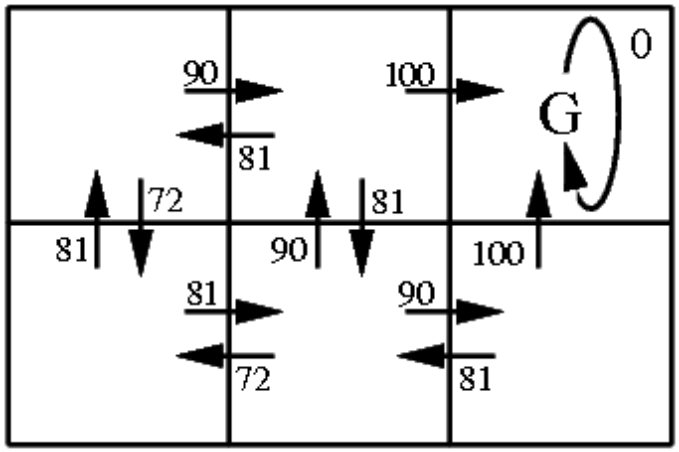
$$\Rightarrow \pi^*(s) = \underset{a}{argmax} Q(s, a)$$

Q Learning

- hence, learning the Q function corresponds to learning the optimal policy π^*
- if the agent learns Q instead of V^*, it will be able to select optimal actions even when it has *no knowledge of* r and δ
- it only needs to consider each available action a in its current state s and chose the action that maximizes $Q(s, a)$
- the value of $Q(s, a)$ for the current state and action summarizes in one value all information needed to determine the discounted cumulative reward that will be gained in the future if a is selected in s

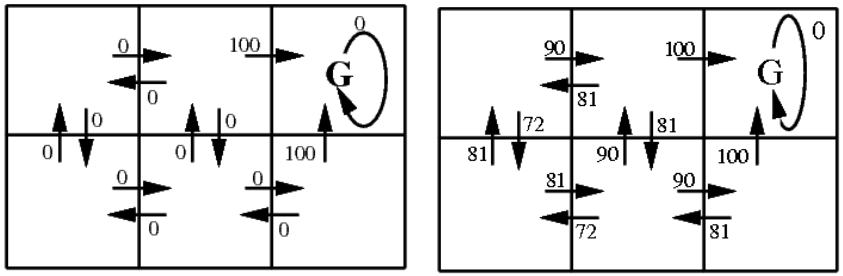
Q Learning





- the right diagram shows the corresponding Q values
- the Q value for each state-action transition equals the r value for this transition plus the V^* value discounted by γ

Q Learning Algorithm



- **key idea:** iterative approximation
- relationship between Q and V^*

$$V^*(s) = \max_{a'} Q(s, a')$$

$$Q(s, a) = r(s, a) + \gamma \max_{a'} Q(\delta(s, a), a')$$

- this recursive definition is the basis for algorithms that use iterative approximation
- the learner's estimate $\hat{Q}(s, a)$ is represented by a large table with a separate entry for each state-action pair

Q Learning Algorithm

Algorithm

For each s, a initialize the table entry $\hat{Q}(s, a)$ to zero
Observe the current state s
Do forever:

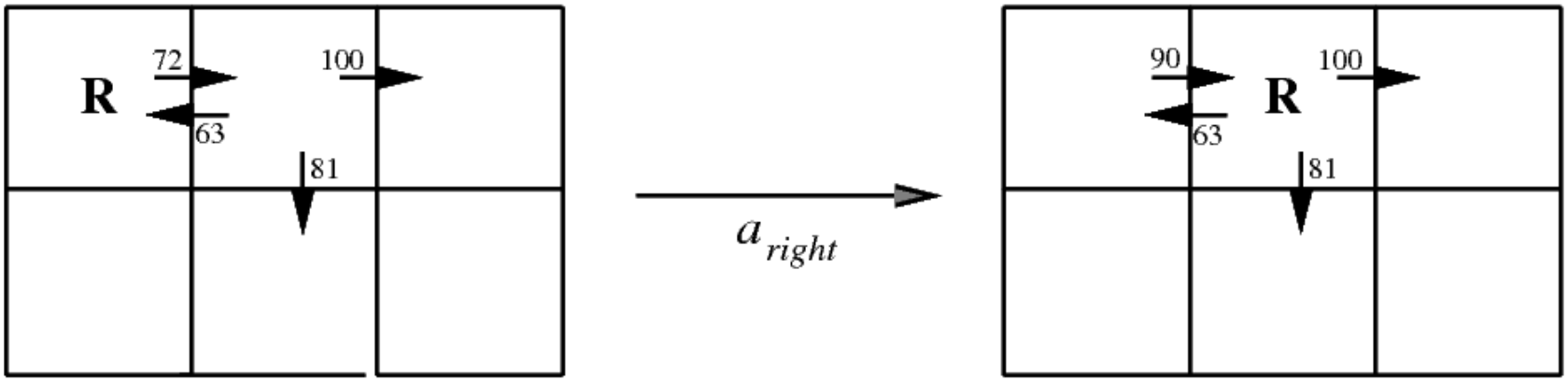
- Select an action a and execute it
- Receive immediate reward r
- Observe new state s'
- Update the table entry for $\hat{Q}(s, a)$ as follows

$$\hat{Q}(s, a) \leftarrow r + \gamma max_{a'} \hat{Q}(s', a')$$

- $s \leftarrow s'$

$\Rightarrow$ Using this algorithm the agent's estimate $\hat{Q}$ converges to the actual Q, provided the system can be modeled as a deterministic Markov decision process, r is bounded, and actions are chosen so that every state-action pair is visited infinitely often.

Illustrative Example

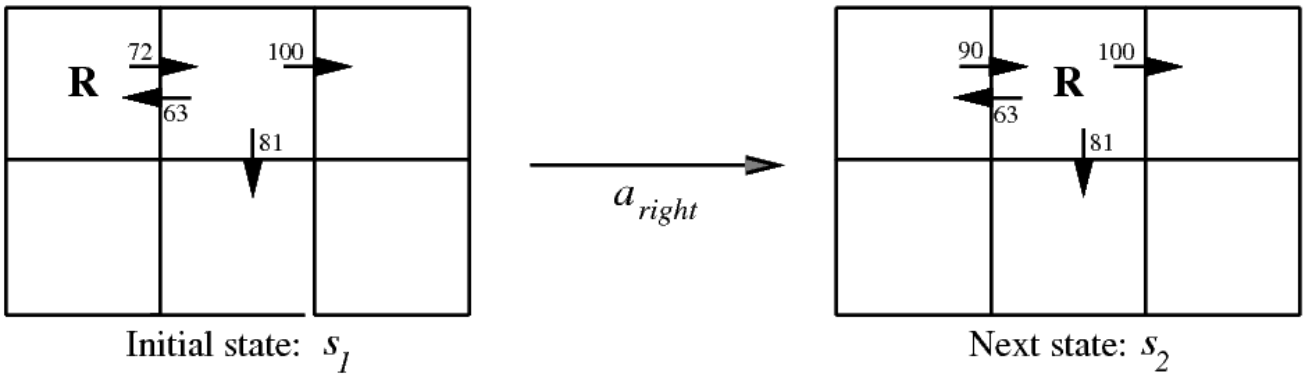


Initial state: s_1 Next state: s_2

$$\hat{Q}(s_1, a_{right}) \leftarrow r + \gamma \cdot \max_{a'} \hat{Q}(s_2, a')$$
$$\leftarrow 0 + 0.9 \cdot \max\{63, 81, 100\}$$
$$\leftarrow 90$$

- the old values are read from our $\hat{Q}$-table, which are about to be updated in each step
- each time the agent moves, Q Learning propagates $\hat{Q}$ estimates *backwards* from the new state to the old and updates the corresponding value in the table

Illustrative Example



Initial state: s_1 Next state: s_2

Table before the move

(s, a)	$\hat{Q}$
$1, \rightarrow$	72
$2, \leftarrow$	63
$2, \rightarrow$	100
$2, \downarrow$	81

Table after the move

(s, a)	$\hat{Q}$
$1, \rightarrow$	**90**
$2, \leftarrow$	63
$2, \rightarrow$	100
$2, \downarrow$	81

Experimentation Strategies

- algorithm does not specify how actions are chosen by the agent
- **obvious strategy:** select action a that maximizes $\hat{Q}(s, a)$
  - risk of overcommitting to actions with high $\hat{Q}$ values during earlier trainings
  - exploration of yet unknown actions is neglected
- **alternative:** probabilistic selection

$$P(a_i|s) = \frac{k^{\hat{Q}(s,a_i)}}{\sum_j k^{\hat{Q}(s,a_j)}}$$

$k > 0$ indicates how strongly the selection favors actions with high $\hat{Q}$ values

k large $\Rightarrow$ exploitation strategy
k small $\Rightarrow$ exploration strategy

Generalizing From Examples

- so far, the target function is represented as an explicit lookup table
- the algorithm performs a kind of rote learning and makes no attempt to estimate the Q value for yet unseen state-action pairs

$\Rightarrow$ unrealistic assumption in large or infinite spaces or when execution costs are very high

- incorporation of function approximation algorithms such as BACKPROPAGATION
  - table is replaced by a neural network using each $\hat{Q}(s, a)$ update as training example (s and a are inputs, $\hat{Q}$ the output)
  - a neural network for each action a

Relationship to Dynamic Programming

- Q Learning is closely related to dynamic programming approaches that solve Markov Decision Processes
- **dynamic programming**
  - assumption that $\delta(s, a)$ and $r(s, a)$ are known
  - focus on how to compute the optimal policy
  - mental model can be explored (no direct interaction with environment)

  $\Rightarrow$ *offline system*
- Q **Learning**
  - assumption that $\delta(s, a)$ and $r(s, a)$ are not known
  - direct interaction inevitable

  $\Rightarrow$ *online system*

Relationship to Dynamic Programming

- relationship is append by considering the Bellman's equation, which forms the foundation for many dynamic programming approaches solving Markov Decision Processes

$$(\forall s \in S) V^*(s) = E[r(s, \pi(s)) + \gamma V^*(\delta(s, \pi(s)))]$$

Advanced Topics

- different updating sequences
- proof of convergence
- non-deterministic rewards and actions
- temporal difference learning

Summary

- Reinforcement learning is an incremental approach to learning where an agent improves its performance by optimizing a reward function
- In contrast to classification learning, a **policy** (strategy to select actions) is learned
- A basic approach to RL is *Q*-Learning where the numerical evaluation function is learned based on a dynamic programming principle

Learning Terminology

Q-Learning, Reinforcement Learning

Supervised Learning	unsupervised learning

Approaches:

Concept / Classification	**Policy Learning**
symbolic	**statistical** / neuronal network
inductive	analytical

Learning Strategy:	**incremental learning from reinforcement**
Data:	**categorial/metric features**
Output:	**action sequence**