Lecture 2: Concept Learning

Version Space, Candidate Elimination, Inductive Bias

Ute Schmid

Cognitive Systems, Applied Computer Science, University of Bamberg
www.uni-bamberg.de/cogsys

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Outline

- Definition of concept learning
- FIND-S
- Version Spaces
- Candidate Elimination
- Inductive Bias
- Summary

Definition of Concept Learning

- **Learning** involves acquiring general concepts from a specific set of training examples D
- Each **concept** c can be thought of as a boolean-valued function defined over a larger set
  (i.e. a function defined over all animals, whose value is true for birds and false for other animals)

$\Rightarrow$ **Concept learning**: Inferring a boolean-valued function from training examples

A Concept Learning Task - Informal

- example target concept
  Enjoy: *"days on which Aldo enjoys his favorite sport"*
- set of example days D, each represented by a set of attributes

Example	*Sky*	*AirTemp*	*Humidity*	*Wind*	*Water*	*Forecast*	*Enjoy*
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

- the task is to learn to predict the value of *Enjoy* for an arbitrary day, based on the values of its other attributes

A Concept Learning Task - Informal

- Hypothesis representation
  - Each hypothesis h consists of a **conjunction of constraints on the instance attributes**, that is, in this case a vector of six attributes
  - Possible constraints:
    - $\Rightarrow$?: any value is acceptable
    - $\Rightarrow$ single required value for the attribute
    - $\Rightarrow$ $\emptyset$: no value is acceptable
  - if some instance x satisfies all the constraints of hypothesis h, then h classifies x as a positive example ($h(x) = 1$)

$\Rightarrow$ **most general** hypothesis: $<?, ?, ?, ?, ?, ? >$
$\Rightarrow$ **most specific** hypothesis: $< \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset >$

A Concept Learning Task - Formal

Given:

- Instances X: Possible days, each described by the attributes
  - *Sky* (with values *Sunny*, *Cloudy* and *Rainy*)
  - *AirTemp* (with values *Warm* and *Cold*)
  - *Humidity* (with values *Normal* and *High*)
  - *Wind* (with values *Strong* and *Weak*)
  - *Water* (with values *Warm* and *Cool*)
  - *Forecast* (with values *Same* and *Change*)
- Hypotheses H where each $h \in H$ is described as a conjunction of constraints on the above attributes
- Target Concept $c : Enjoy : X \Rightarrow \{0, 1\}$
- Training examples D: positive and negative examples of the table above

Determine:

- A hypothesis $h \in H$ such that $(\forall x \in X)[h(x) = c(x)]$

A Concept Learning Task - Example

- **example hypothesis** $h_e = < Sunny, ?, ?, ?, Warm, ? >$
  - $\Rightarrow$ According to h_e Aldo enjoys his favorite sport whenever the *sky* is sunny and the *water* is warm (independent of the other weather conditions!)

example 1: $< \mathbf{Sunny}, Warm, Normal, Strong, \mathbf{Warm}, Same >$

This example satisfies h_e, because the *sky* is sunny and the *water* is warm. Hence, Aldo would enjoy his favorite sport on this day.

example 4: $< \mathbf{Sunny}, Warm, High, Normal, \mathbf{Cool}, Change >$

This example does **not** satisfy h_e, because the *water* is cool. Hence, Aldo would not enjoy his favorite sport on this day.

$\Rightarrow$ h_e is **not** consistent with the training examples D

Concept Learning as Search

- concept learning as search through the space of hypotheses H (implicitly defined by the hypothesis representation) with the goal of finding the hypothesis that best fits the training examples
- most practical learning tasks involve very large, even **infinite hypothesis spaces**
- many concept learning algorithms organize the search through the hypothesis space by relying on the **general-to-specific ordering**

FIND-S

- exploits general-to-specific ordering
- finds a maximally specific hypothesis h consistent with the observed training examples D

FIND-S Algorithm

1. Initialize h to the most specific hypothesis in H
2. For each positive training instance x
   - if the constraint a_i is satisfied by x
     then do nothing
     else replace a_i with the next more general constraint satisfied by x
3. Output hypothesis h

FIND-S – Example

- Initialize $h \leftarrow \; < \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset >$
- example 1: $< \textbf{Sunny}, \textbf{Warm}, \textbf{Normal}, \textbf{Strong}, \textbf{Warm}, \textbf{Same} >$

  $\Rightarrow \; h \leftarrow \; < Sunny, Warm, Normal, Strong, Warm, Same >$

- example 2: $< Sunny, Warm, \textbf{High}, Strong, Warm, Same >$

  $\Rightarrow \; h \leftarrow \; < Sunny, Warm, ?, Strong, Warm, Same >$

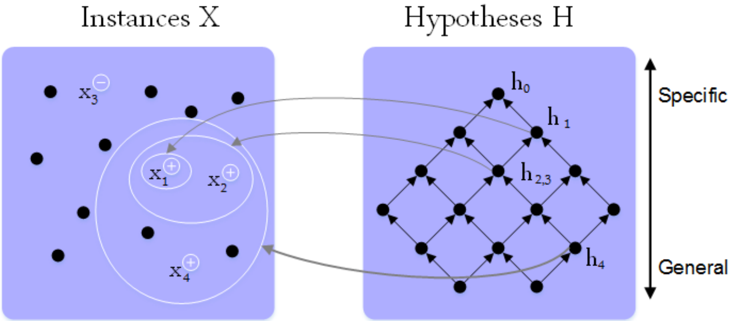
- example 3: $< Rainy, Cold, High, Strong, Warm, Change >$

  This example can be omitted because it is negative.
  Notice that the current hypothesis is already consistent with this example, because it correctly classifies it as negative!

- example 4: $< Sunny, Warm, High, Strong, \textbf{Cool}, \textbf{Change} >$

  $\Rightarrow \; h \leftarrow \; < Sunny, Warm, ?, Strong, ?, ? >$

FIND-S – Example



$$h_0 = \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$$

$$x_1 = \langle Sunny, Warm, Normal, Strong, Warm, Same \rangle, +$$
$$h_1 = \langle Sunny, Warm, Normal, Strong, Warm, Same \rangle$$

$$x_2 = \langle Sunny, Warm, High, Strong, Warm, Same \rangle, +$$
$$h_2 = \langle Sunny, Warm, ?, Strong, Warm, Same \rangle$$

$$x_3 = \langle Rainy, Cold, High, Strong, Warm, Change \rangle, -$$
$$h_3 = \langle Sunny, Warm, ?, Strong, Warm, Same \rangle$$

$$x_4 = \langle Sunny, Warm, High, Strong, Cool, Change \rangle, +$$
$$h_4 = \langle Sunny, Warm, ?, Strong, ?, ? \rangle$$

Remarks on FIND-S

- in each step, h is consistent with the training examples observed up to this point
- unanswered questions:
  - Has the learner converged to the correct target concept?

    No way to determine whether FIND-S found the only consistent hypothesis h or whether there are many other consistent hypotheses as well
  - Why prefer the most specific hypothesis?
  - Are the training examples consistent?

    FIND-S is only correct if D itself is consistent. That is, D has to be free of classification errors.
  - What if there are several maximally specific consistent hypotheses?

CANDIDATE-ELIMINATION

- CANDIDATE-ELIMINATION addresses several limitations of the FIND-S algorithm
- **key idea**: description of the set of all hypotheses consistent with D without explicitly enumerating them
- performs poorly with noisy data
- useful conceptual framework for introducing fundamental issues in machine learning

Version Spaces

- to incorporate the key idea mentioned above, a compact representation of all consistent hypotheses is necessary
- **Version space** $VS_{H,D}$, with respect to hypothesis space H and training data D, is the subset of hypotheses from H consistent with D.

$$VS_{H,D} \equiv \{h \in H | Consistent(h, D)\}$$

- $VS_{H,D}$ can be represented by the **most general** and the **most specific** consistent hypotheses in form of **boundary sets** within the partial ordering

Version Spaces

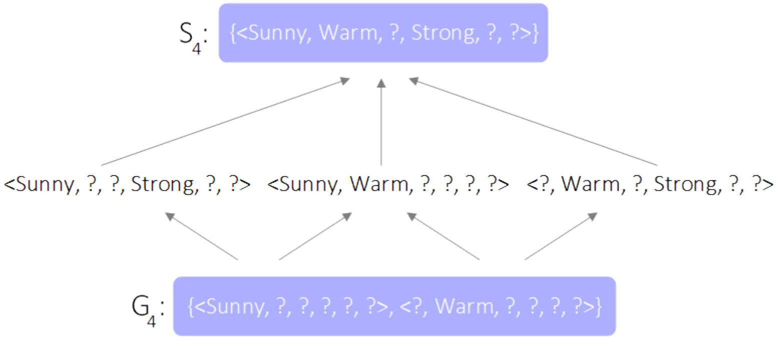
- The **general boundary set** G, with respect to hypothesis space H and training data D, is the set of maximally general members of H consistent with D.

  $G \equiv \{g \in H | Consistent(g, D) \wedge (\neg \exists g' \in H)[(g' >_g g) \wedge Consistent(g', D)]\}$

- The **specific boundary set** S, with respect to hypothesis space H and training data D, is the set of minimally general (i.e., maximally specific) members of H consistent with D.

  $S \equiv \{s \in H | Consistent(s, D) \wedge (\neg \exists s' \in H)[(s >_g s') \wedge Consistent(s', D)]\}$

Version Spaces



Candidate Elimination Algorithm

Initialize G to the set of maximally general hypotheses in H
Initialize S to the set of maximally specific hypotheses in H
For each training example $d \in D$, do

- **If d is a positive example**
  - Remove from G any hypothesis inconsistent with d
  - For each hypothesis s in S that is inconsistent with d
    - Remove s from S
    - Add to S all minimal generalizations h of s such that h is consistent with d and some member of G is more general than h
    - Remove from S any hypothesis that is more general than another hypothesis in S
- **If d is a negative example**
  - Remove from S any hypothesis inconsistent with d
  - For each hypothesis g in G that is inconsistent with d
    - Remove g from G
    - Add to G all minimal specializations h of g such that h is consistent with d and some member of S is more specific than h
    - Remove from G any hypothesis that is less general than another hypothesis in G

Illustrative Example

- Initialization of the Boundary sets
  - $G_0 \leftarrow \{\langle ?, ?, ?, ?, ?, ? \rangle\}$
  - $S_0 \leftarrow \{\langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle\}$

- example 1: $\langle \textbf{Sunny}, \textbf{Warm}, \textbf{Normal}, \textbf{Strong}, \textbf{Warm}, \textbf{Same} \rangle$

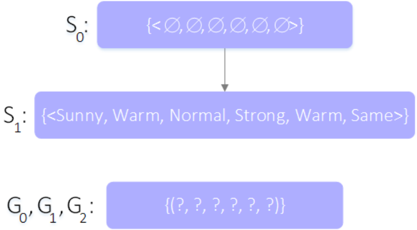
  S is overly specific, because it wrongly classifies example 1 as false. So S has to be revised by moving it to the **least more general hypothesis** that covers example 1 and is **still more special** than another hypothesis in G .

  $\Rightarrow\ S_1 = \{\langle Sunny, Warm, Normal, Strong, Warm, Same \rangle\}$
  $\Rightarrow\ G_1 = G_0$

- example 2: $\langle Sunny, Warm, \textbf{High}, Strong, Warm, Same \rangle$

  $\Rightarrow\ S_2 = \{\langle Sunny, Warm, ?, Strong, Warm, Same \rangle\}$
  $\Rightarrow\ G_2 = G_1 = G_0$

Illustrative Example



Training Examples:

1. $\langle Sunny, Warm, Normal, Strong, Warm, Same \rangle, EnjoySport = Yes$
2. $\langle Sunny, Warm, High, Strong, Warm, Same \rangle, EnjoySport = Yes$

Illustrative Example

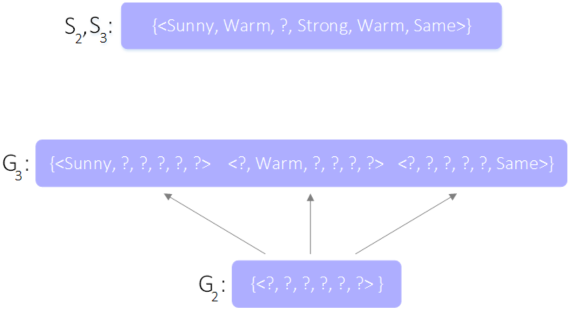
- example 3: $\langle \textbf{Rainy}, \textbf{Cold}, \textit{High}, \textit{Strong}, \textit{Warm}, \textbf{Change} \rangle$

  G is overly general, because it wrongly classifies example 3 as true. So G has to be revised by moving it to the **least more specific hypotheses** that covers example 3 and is **still more general** than another hypothesis in S.

  There are several alternative minimally more specific hypotheses.

$\Rightarrow$ $S_3 = S_2$
$\Rightarrow$ $G_3 = \{\langle Sunny, ?, ?, ?, ?, ? \rangle, \langle ?, Warm, ?, ?, ?, ? \rangle, \langle ?, ?, ?, ?, ?, Same \rangle\}$

Illustrative Example



Trainings Examples:

3.$\langle Rainy, Cold, High, Strong, Warm, Change\rangle, EnjoySport = No$

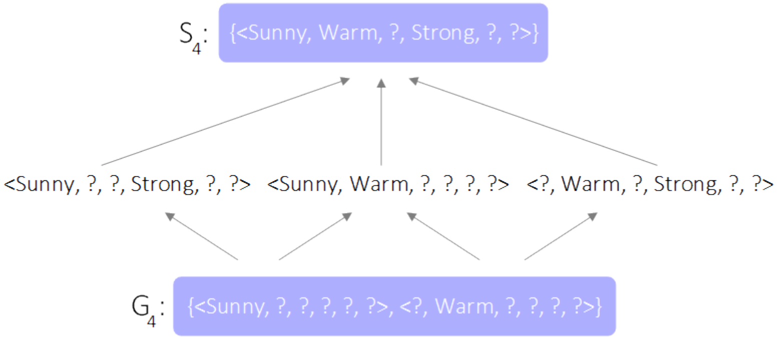
Illustrative Example

- example 4: $\langle Sunny, Warm, High, Strong, \textbf{Cool}, \textbf{Change}\rangle$

  $\Rightarrow\ S_4 = \{\langle Sunny, Warm, ?, Strong, ?, ?\rangle\}$

  $\Rightarrow\ G_4 = \{\langle Sunny, ?, ?, ?, ?, ?\rangle, \langle ?, Warm, ?, ?, ?, ?\rangle\}$

Illustrative Example



Remarks

- Will the algorithm converge to the correct hypothesis?
  - convergence is assured provided there are no errors in D and the H includes the target concept
  - G and S contain only the same hypothesis
- How can partially learned concepts be used?
  - some unseen examples can be classified unambiguously as if the target concept had been fully learned
    - **positive** iff it satisfies every member of S
    - **negative** iff it doesn't satisfy any member of G
  - otherwise an instance x is classified by majority (if possible)

Inductive Bias

- fundamental property of inductive learning
  - a learner that makes no a priori assumptions regarding the identity of the target concept has no rational basis for classifying unseen examples
  - e.g., learning the **EnjoySport** concept was based on the assumption that the target concept could be represented as a conjunction of attribute values
- inductive bias $\approx$ policy by which the learner generalizes beyond the observed training data to infer the classification of new instances

Inductive Bias

- Consider a concept learning algorithm L for the set of instances X. Let c be an arbitrary concept defined over X, and $D_c = \{< x, c(x) >\}$ an arbitrary set of training examples of c. Let $L(x_i, D_c)$ denote the classification assigned to the instance x_i by L after training on the data D_c. The **inductive bias** of L is any minimal set of assertions B such that

$$(\forall x_i \in X)[(B \wedge D_c \wedge x_i) \vdash L(x_i, D_c)]$$

Kinds of Inductive Bias

- **Restriction Bias** (aka Language Bias)
  - entire H is searched by learning algorithm
  - hypothesis representation **not expressive enough** to encompass all possible concepts
  - e.g. CANDIDATE-ELIMINATION: for the hypothesis language used in the "enjoy"-example H only includes conjunctive concepts
- **Preference Bias** (aka Search Bias)
  - hypothesis representation encompasses all possible concepts
  - learning algorithm does not consider each possible hypothesis
  - e.g. use of heuristics, greedy strategies

$\Rightarrow$ Preference Bias more desirable, because it assures

$$(\exists h \in H)[(\forall x \in X)[h(x) = c(x)]]$$

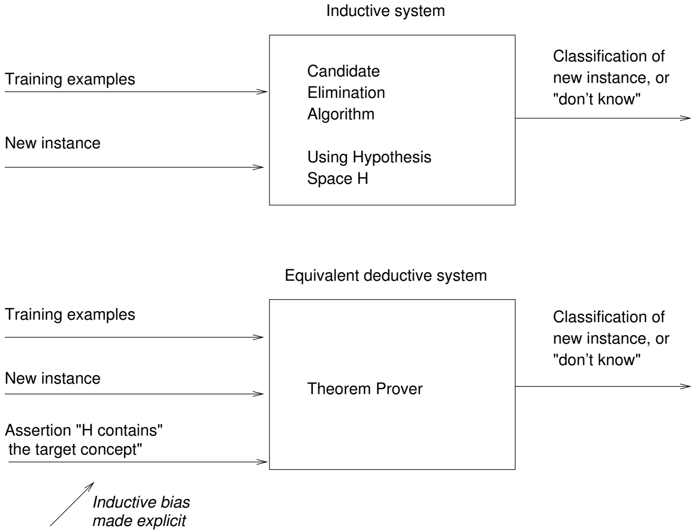
An Unbiased Learner

- an unbiased $H = 2^{|X|}$ would contain every teachable function
- for such a H,
  - G would always contain the **negation of the disjunction of observed negative examples**
  - S would always contain the **disjunction of the observed positive examples**
- hence, only observed examples will be classified correctly

$\Rightarrow$ in order to converge to a single target concept, every $x \in X$ has to be in D

$\Rightarrow$ the learning algorithm is unable to generalize beyond observed training data

Inductive System vs. Theorem Prover



Summary

- Concept learning is defined as learning a boolean-valued function from training examples.
- For some hypothesis spaces, hypotheses can be ordered by generality. This allows for efficient representation as version spaces.
- The candidate-elimination algorithm exploits the general to specific ordering of hypotheses in version space.
- There is no bias-free learning.
- The hypothesis language defines a restriction bias (maybe some problems cannot be represented and therefore not learned).
- The search-strategy defines a preference bias (the "correct" hypothesis might be found, if the search strategy is "suitable").
- If the inductive bias of a learner were represented explicitely, induction could be posed as a deduction problem: deriving a classification of a new example from bias and training examples.

FIND-S / CANDIDATE ELIMINATION

Supervised Learning	unsupervised learning

Approaches:

Concept / Classification	Policy Learning
symbolic	statistical / neuronal network
inductive	analytical

Learning Strategy:
⇒ **learning from examples**