Lecture 6: Inductive Logic Programming

Sequential Covering, FOIL, Inverted Resolution, EBL

Ute Schmid

Cognitive Systems, Applied Computer Science, University of Bamberg
`www.uni-bamberg.de/cogsys`

Last change: November 19, 2019

Motivation

- it is useful to learn the target function as a set of if-then-rules
    - one of the most expressive and human readable representations
    - e.g. decision trees
- **Inductive Logic Programming (ILP):**
    - rules are learned directly
    - designed to learn *first-order rules* (i.e. including variables)
    - *sequential covering* to incrementally grow the final set of rules
- PROLOG programs are sets of first-order rules

⇒ a general-purpose method capable of learning such rule sets can be viewed as an algorithm for automatically inferring PROLOG programs

Outline

- Sequential Covering
    - General to Specific Beam Search
    - Learn One Rule
    - Performance Measures
    - Sequential vs. Simultaneous Covering
- First-Order Rules
- FOIL
    - Hypothesis Space
    - Generating Candidate Specializations
    - Example
    - FOIL Gain
- Induction as Inverted Deduction
    - Inverse Resolution
    - Generalization, θ-Subsumption, Entailment
- Explanation-based Learning (EBL)
- Summary

Examples

- Propositional Logic:

```
IF Humidity=normal AND Outlook=sunny
THEN PlayTennis=yes
IF Humidity=normal AND Temperature=mild AND Wind=weak
THEN PlayTennis=yes
```

```
playTennis :- humidity(normal), outlook(sunny).
playTennis :- humidity(normal), temperature(mild),
              wind(weak).
```

- First Order Logic:

```
IF Parent(x,y) THEN Ancestor(x,y)
IF Parent(x,z) AND Ancestor(z,y) THEN Ancestor(x,y)
```

```
ancestor(X,Y) :- parent(X,Y).
ancestor(X,Y) :- parent(X,Z), ancestor(Z,Y).
```

Sequential Covering

- **basic strategy**: learn one rule, remove the data it covers, then iterate this process
- one of the most widespread approaches to learn a disjunctive set of rules (each rule itself is conjunctive)
- subroutine **LEARN-ONE-RULE**
  - accepts a set of positive and negative examples as input and outputs a single rule that covers many of the positive and few of the negative examples
  - **high accuracy**: predictions should be correct
  - **low coverage**: not necessarily predictions for each example
- performs a greedy search without backtracking

  ⇒ no guarantee to find the smallest or best set of rules

Sequential Covering

Algorithm

```
SEQUENTIAL-COVERING(Target_attribute, Attributes, Examples, Threshold)
    Learned_Rules ← {}
    Rule ← LEARN-ONE-RULE(Target_attribute, Attributes, Examples)
    While PERFORMANCE(Rule, Examples) > Threshold, Do
        Learned_rules ← Learned_rules + Rule
        Examples ← Examples − { examples correctly classified by Rule}
        Rule ← LEARN-ONE-RULE(Target_attribute, Attributes, Examples)
    Learned_rules ← sort Learned_rules accord to PERFORMANCE over Examples
    return Learned_rules
```

General to Specific Beam Search

- **question**: How shall LEARN-ONE-RULE be designed to meet the needs of the sequential covering algorithm?
- organize the search through H analogous to ID3
  - **but** follow only the most promising branch in the tree at each step
  - begin by considering the most general rule precondition (i.e. empty test)
  - then greedily add the attribute test that most improves rule performance over the training examples
  - unlike to ID3, this implementation follows only a single descendant at each search step rather than growing a sub-tree that covers all possible values of the selected attribute

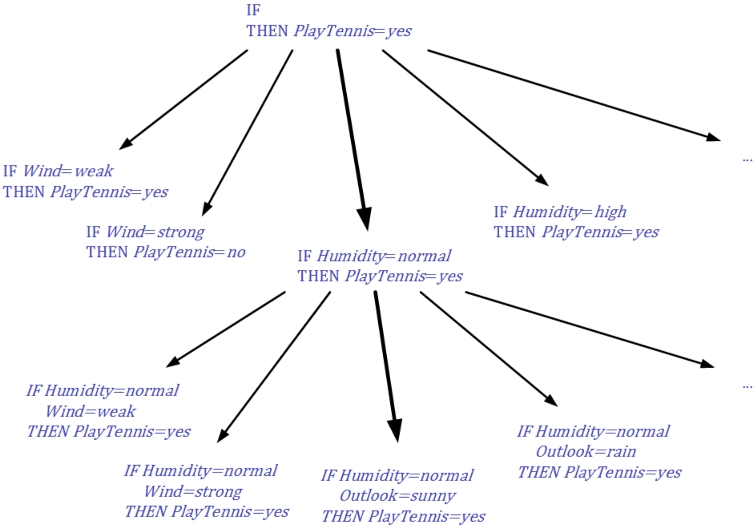
General to Specific Beam Search

- **so far** a local greedy search (analogous to hill-climbing) is employed
  - danger of suboptimal results
  - susceptible to the typical hill-climbing problems

⇒ extension to **beam search**

- ⇒ algorithm maintains a list of the k best candidates at each step
- ⇒ at each step, descendants are generated for each of the k candidates and the resulting set is again reduced to the k most promising candidates

General to Specific Beam Search



IF
THEN *PlayTennis=yes*

IF *Wind=weak*
THEN *PlayTennis=yes*

IF *Wind=strong*
THEN *PlayTennis=no*

IF *Humidity=normal*
THEN *PlayTennis=yes*

IF *Humidity=high*
THEN *PlayTennis=yes*

...

IF Humidity=normal
Wind=weak
THEN PlayTennis=yes

IF Humidity=normal
Wind=strong
THEN PlayTennis=yes

IF Humidity=normal
Outlook=sunny
THEN PlayTennis=yes

IF Humidity=normal
Outlook=rain
THEN PlayTennis=yes

...

LEARN-ONE-RULE

LEARN-ONE-RULE(*Target_attribute*, *Attributes*, *Examples*, *k*)

Returns a single rule that covers some of the Examples. Conducts a general to specific greedy beam search for the best rule, guided by the PERFORMANCE metric.

- Initialize *Best_hypothesis* to the most general hypothesis $\emptyset$
- Initialize *Candidate_hypotheses* to the set $\{$*Best_hypothesis*$\}$
- While *Candidate_hypotheses* is not empty, Do
  1. *Generate the next more specific Candidate_hypotheses*
     - *New_Candidate_hypotheses* ← new generated and specialized candidates
  2. *Update Best_hypotheses*
     - Select hypothesis *h* from *New_Candidate_hypotheses* with best PERFORMANCE over Examples
     - IF PERFORMANCE of *h* > PERFORMANCE of *Best_hypothesis* THEN set *h* as new *Best_hypothesis*
  3. *Update Candidate_hypotheses*
     - *Candidate_hypotheses* ← the *k* best members of *New_Candidate_hypotheses*

$\hookrightarrow$

LEARN-ONE-RULE cont.

- Return a rule of the form

  "IF *Best_hypothesis* THEN *prediction*"
  where *prediction* is the most frequent value of *Target_attribute* among those *Examples* that match *Best_hypothesis*.

Example: Learn One Rule

- Learn *one* rule that covers a certain amount of positive examples
- with high accuracy
- remove the covered positive examples

Example

(s1)	Sky = sunny	(s2)	Sky = rainy
(a1)	AirTemp = warm	(a2)	AirTemp = cold
(h1)	Humidity = normal	(h2)	Humidity = high
(w1)	Water = warm	(w2)	Water = cool

Example cont.

- Assume $k = 4$
- Current most specific hypotheses: s1, s2, a1, a2, h1, h2, w1, w2
- Assume best possible hypotheses wrt performance P: s1, a2, h1, w1
- Generate new candidate hypotheses, e.g. by specializing s1:

s1 & s1 (duplicate)
s1 & s2 (inconsistent)
s1 & a1
s1 & a2
s1 & h1
...

Performance Measures

- Relative Frequency (numbers of correctly classified examples by all examples)

$$\frac{n_c}{n}$$

- Entropy S as set of examples that match precondition, p_i proportion of examples from S for which the target function takes the i-th value

$$-Entropy(S) = \sum_{i=1}^{c} p_i log_2 p_i$$

Sequential vs. Simultaneous Covering

- **sequential covering:**
  - learn just one rule at a time, remove the covered examples and repeat the process on the remaining examples
  - many search steps, making independent decisions to select each precondition for each rule

- **simultaneous covering:**
  - ID3 learns the entire set of disjunctive rules simultaneously as part of a single search for a decision tree
  - fewer search steps, because each choice influences the preconditions of all rules

$\Rightarrow$ Choice depends of how much data is available

- plentiful → sequential covering (more steps supported)
- scarce → simultaneous covering (decision sharing effective)

Differences in Search

- **generate-then-test:**
  - search through all syntactically legal hypotheses
  - generation of the successor hypotheses is only based on the syntax of the hypothesis representation
  - training data is considered after generation to choose among the candidate hypotheses
  - each training example is considered several times

  ⇒ impact of noisy data is minimized

- **example driven:**
  - individual training examples constrain the generation of hypotheses
  - e.g. FIND-S, CANDIDATE ELIMINATION

  ⇒ search can easily be misled

Learning First-Order Rules

- propositional expressions do not contain variables and are therefore less expressive than first-order expressions
- no general way to describe essential relations among the values of attributes
- **Literals:** We refer to atomic formulas also as **atoms**. Positive and negative atoms ($P, \neg P$) are called **positive/negative literals**
  e.g. $parent(x, y)$ or $\neg parent(x, y) \ldots$
- **Clauses:** A **clause** is a *disjunction* of positive and negative **literals**.
  e.g. $mother\_of(x, y) \vee father\_of(z, y)$

Learning First-Order Rules

Now we consider learning first-order rules (Horn Theories)

- a **Horn Clause** is a clause containing at most *one positive literal*
- expression of the form:

$$\begin{aligned} & H \vee \neg L_1 \vee \neg L_2 \vee \ldots \vee \neg L_n \\ & \qquad \Longleftrightarrow H \leftarrow (L_1 \wedge L_2 \wedge \ldots \wedge L_n) \\ & \qquad\qquad \Longleftrightarrow \text{IF } (L_1 \wedge L_2 \wedge \ldots \wedge L_n) \text{ THEN } H \end{aligned}$$

- FOL terminology see *Intelligente Agenten*

FOIL (Quinlan, 1990)

- natural extension of SEQUENTIAL-COVERING and LEARN-ONE-RULE
- outputs sets of first-order rules similar to Horn Clauses with two exceptions
  1. **more restricted**, because literals are not permitted to contain function symbols
  2. **more expressive**, because literals in the body can be negated
- differences between FOIL and earlier algorithms:
  - seeks only rules that predict when the target literal is *True*
  - conducts a simple hill-climbing search instead of beam search

FOIL

Algorithm

```
FOIL(Target_predicate, Predicates, Examples)
  Pos ← those Examples for which the Target_predicate is True
  Neg ← those Examples for which the Target_predicate is False
  Learned_rules ← {}
  while Pos, Do
      NewRule ← the rule that predicts Target_predicate with no precondition
      NewRuleNeg ← Neg
      while NewRuleNeg, Do
          Candidate_literals ← generate new literals for NewRule, based on Predicates
          Best_literal ← $\mathrm{argmax}_{L \in Candidate\_literals} FoilGain(L, NewRule)$
          add Best_literal to preconditions of NewRule
          NewRuleNeg ← subset of NewRuleNeg that satisfies NewRule preconditions
      Learned_rules ← Learned_rules + NewRule
      Pos ← Pos − { members of Pos covered by NewRule}
  Return Learned_rules
```

FOIL Hypothesis Space

- **outer loop (set of rules):**
  - specific-to-general search
  - initially, there are no rules, so that each example will be classified negative (most specific)
  - each new rule raises the number of examples classified as positive (more general)
  - disjunctive connection of rules
- **inner loop (preconditions for one rule):**
  - general-to-specific search
  - initially, there are no preconditions, so that each example satisfies the rule (most general)
  - each new precondition raises the number of examples classified as negative (more specific)
  - conjunctive connection of preconditions

Generating Candidate Specializations

- current rule:

  $P(x_1, x_2, ..., x_k) \leftarrow L_1 ... L_n$ where

  $\Rightarrow$ $L_1 ... L_n$ are the preconditions and

  $\Rightarrow$ $P(x_1, x_2, ..., x_k)$ is the head of the rule

- FOIL generates candidate specializations by considering new literals L_{n+1} that fit one of the following forms:
  - $Q(v_1, ..., v_r)$ where $Q \in Predicates$ and the v_i are new or already present variables (at least one v_i must already be present)
  - $Equal(x_j, x_k)$ where x_j and x_k are already present in the rule
  - the negation of either of the above forms

Training Data Example

Examples	Background Knowledge
GrandDaughter(Victor, Sharon)	Female(Sharon)
¬ GrandDaugther(Tom, Bob)	Father(Sharon, Bob)
¬ GrandDaughter(Victor, Victor)	Father(Tom, Bob)
	Father(Bob, Victor)

Remark: We assume closed world assumption for the background knowledge (e.g., not Female(Bob)). For examples we only consider the given target predicates.

FOIL Example

Example

GrandDaughter(x,y) $\leftarrow$

Candidate additions to precondition:

Equal(x,y), Female(x), Female(y), Father(x,y), Father(y,x), Father(x,z),Father(z,x), Father(z,y), and the negations to these literals

Assume greedy selection of Father(y,z):

GrandDaughter(x,y) $\leftarrow$ **Father(y,z)**

Candidate additions:

the ones from above and Female(z), Equal(z,y), Father(z,w), Father(w,z), and their negations

Learned Rule:

GrandDaughter(x,y) $\leftarrow$ **Father(y,z)** $\wedge$ **Father(z,x)** $\wedge$ **Female(y)**

FOIL Gain

$$FoilGain(L, R) \equiv t \left(log_2 \frac{p_1}{p_1 + n_1} - log_2 \frac{p_0}{p_0 + n_0} \right)$$

with

- L as new literal introduced in rule R to gain new rule R'
- t as number of positive bindings of rule R which are still covered by R'
- p_1 as number of positive bindings of rule R' and n_1 as number of negative bindings
- p_0 as number of positive bindings of rule R and n_0 as number of negative bindings

Remark: Bindings are the number of instantiations of the variables by constants. A binding is positive if the instantiated rule covers a positive example.

Learning Recursive Rule Sets

- Extend FOIL such that the target predicate can also be included in the preconditions with the same restrictions to variables as before.
- Problem: rule sets that produce infinite recursions
- FOIL uses a generate-and-test strategy; alternatively recursive rule sets can be learned by analytical methods
  (see lecture inductive programming)

```
ancestor(X,Y) :- parent(X,Y).
ancestor(X,Y) :- parent(X,Z), ancestor(Z,Y).
```

Induction as Inverted Deduction

- **observation:** induction is just the inverse of deduction
- in general, machine learning involves building theories that explain the observed data
- Given some data D and some background knowledge B, learning can be described as generating a hypothesis h that, together with B, explains D.

$$(\forall < x_i, f(x_i) > \in D)(B \wedge h \wedge x_i) \vdash f(x_i)$$

- the above equation casts the learning problem in the framework of deductive inference and formal logic

Induction as Inverted Deduction

- **features of inverted deduction:**
  - subsumes the common definition of learning as finding some general concept
  - background knowledge allows a more rich definition of when a hypothesis h is said to "fit" the data
- **practical difficulties:**
  - noisy data makes the logical framework completely lose the ability to distinguish between truth and falsehood
  - search is intractable
  - background knowledge often increases the complexity of H

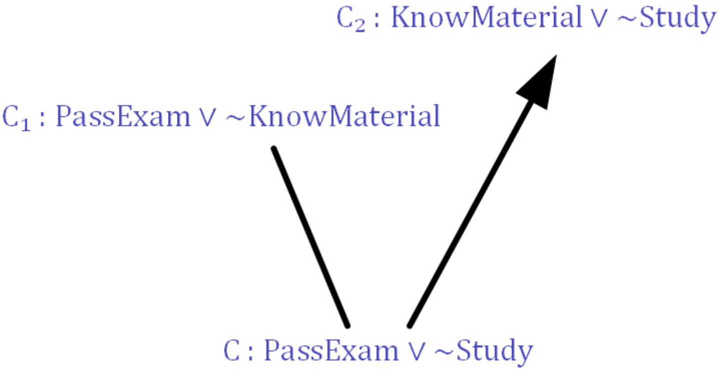
Inverting Resolution

- **resolution** is a general method for automated deduction
- complete and sound method for deductive inference
- see *Intelligente Agenten*
- **Inverse Resolution Operator (propositional form):**
  1. Given initial clause C_1 and C, find a literal L that occurs in C_1 but not in clause C.
  2. Form the second clause C_2 by including the following literals

$$C_2 = (C - (C_1 - \{L\})) \cup \{\neg L\}$$

- inverse resolution is not deterministic

Inverting Resolution



$C_2 : \text{KnowMaterial} \vee \sim\text{Study}$

$C_1 : \text{PassExam} \vee \sim\text{KnowMaterial}$

$C : \text{PassExam} \vee \sim\text{Study}$

Inverting Resolution

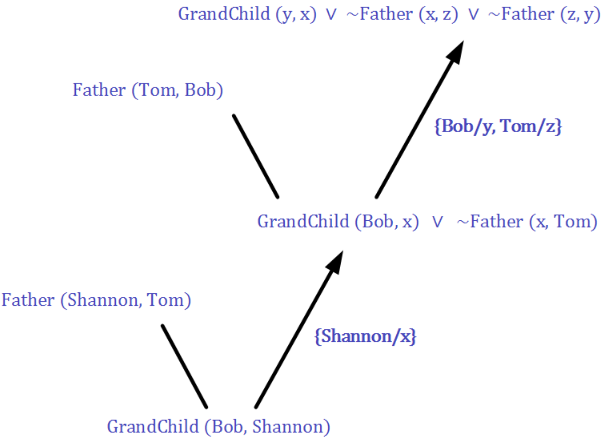
- **Inverse Resolution Operator (first-order form):**
  - resolution rule:
    1. Find a literal L_1 from clause C_1, literal L_2 from clause C_2, and substitution θ such that $L_1\theta = \neg L_2\theta$
    2. Form the resolvent C by including all literals from $C_1\theta$ and $C_2\theta$, except for $L_1\theta$ and $\neg L_2\theta$. That is,
       $C = (C_1 - \{L_1\})\theta \cup (C_2 - \{L_2\})\theta$
  - analytical derivation of the inverse resolution rule:

$$C = (C_1 - \{L_1\})\theta_1 \cup (C_2 - \{L_2\})\theta_2 \text{ where } \theta = \theta_1\theta_2$$

$$C - (C_1 - \{L_1\})\theta_1 = (C_2 - \{L_2\})\theta_2 \text{ where } L_2 = \neg L_1\theta_1\theta_2^{-1}$$

$$\Rightarrow \; C_2 = (C - (C_1 - \{L_1\})\theta_1)\theta_2^{-1} \cup \{\neg L_1\theta_1\theta_2^{-1}\}$$

Inverting Resolution



GrandChild (y, x) ∨ ~Father (x, z) ∨ ~Father (z, y)

Father (Tom, Bob)

{Bob/y, Tom/z}

GrandChild (Bob, x) ∨ ~Father (x, Tom)

Father (Shannon, Tom)

{Shannon/x}

GrandChild (Bob, Shannon)

$D = \{GrandChild(Bob, Shannon)\}$
$B = \{Father(Shannon, Tom), Father(Tom, Bob)\}$

Remarks

- ILP based on restricted variants of inverted resolution was introduced mainly by Muggleton
- Inverse resolution leads to combinatorial explosion of candidate hypotheses
  - many possibilities to combine hypotheses with background- knowledge in order to generate more specific hypotheses

- Other techniques:
  - θ-**Subsumption** (used by *GOLEM*)
    replace terms by variables (intervened unification)
  - **inverse entailment** (used by *PROGOL*)
    generates just a single more specific hypothesis that entails the observed data

Generalization, θ-Subsumption, Entailment

- interesting to consider the relationship between the *more_general_than* relation and inverse entailment
- *more_general_than*:
  $h_j \geq_g h_k$ iff $(\forall x \in X)[h_k(x) \rightarrow h_j(x)]$. A hypothesis can also be expressed as $c(x) \leftarrow h(x)$.
- $\theta -$ *subsumption*:
  Consider two clauses C_j and C_k, both of the form $H \vee L_1 \vee ... \vee L_n$, where H is a positive literal and the L_i are arbitrary literals. Clause C_j is said to $\theta -$ *subsume* clause C_k iff $(\exists\theta)[C_j\theta \subseteq C_k]$.
- *entailment*:
  Consider two clauses C_j and C_k. Clause C_j is said to *entail* clause C_k (written $C_j \vdash C_k$) iff C_k follows deductively from C_j.

Generalization, θ-Subsumption, Entailment

- if $h_1 \geq_g h_2$ then $C_1 : c(x) \leftarrow h_1(x)$ θ-subsumes $C_2 : c(x) \leftarrow h_2(x)$
- furthermore, θ-subsumption can hold even when the clauses have different heads

  $A : Mother(x, y) \leftarrow Father(x, z) \wedge Spouse(z, y)$
  $B : Mother(x, L) \leftarrow Father(x, B) \wedge Spouse(B, y) \wedge Female(x)$

  $A\theta \subseteq B$ if we choose $\theta = \{y/L, z/B\}$
- θ-subsumption is a special case of entailment

  $A : Elephant(father\_of(x)) \leftarrow Elephant(x)$
  $B : Elephant(father\_of(father\_of(y))) \leftarrow Elephant(y)$

  $A \vdash B$, but $\neg\exists\theta[A\theta \subseteq B]$

Generalization, θ-Subsumption, Entailment

- Generalization is a special case of θ-Subsumption
- θ-Subsumption is a special case of entailment
- In its most general form, inverse entailment produces intractable searches
- θ-Subsumption provides a convenient notion that lies midway between generalization and entailment!

Explanation-Based Learning

- Using prior knowledge and deductive reasoning to augment information given by trainings examples
- $\hookrightarrow$ explanation-based learning (EBL)
  e.g., Mitchell et al., 1986
- Different approaches: background-knowledge is or is not complete and correct
- **Reach more accuracy with less examples!**
- For example, applied in inductive programming (Dialogs, Igor)
- Related to learning on the knowledge level and analogy-based learning

Inductive Learning

Given:

- Instance space X
- Hypothesis space H
- Training examples D of some target function f.

$$D = \{\langle x_1, f(x_1)\rangle, \ldots \langle x_n, f(x_n)\rangle\}$$

Determine:

- A hypothesis from H consistent with training examples D.

Analytical Learning

Given:

- Instance space X
- Hypothesis space H
- Training examples D of some target function f.

$$D = \{\langle x_1, f(x_1)\rangle, \ldots \langle x_n, f(x_n)\rangle\}$$

- *Domain theory B for explaining training examples*

Determine:

- A hypothesis from H consistent with both the training examples D and domain theory B.

We say

- B "explains" $\langle x, f(x)\rangle$ if $x + B \vdash f(x)$
- B is "consistent with" h if $B \not\vdash \neg h$

SafeToStack(x,y) Learning Problem

Given:

- Instances: pairs of physical objects
  Hypotheses: Sets of Horn clause rules, e.g.,
  $SafeToStack(x, y) \leftarrow Volume(x, vx) \wedge Type(y, Box)$
- Training Examples: typical example is

$$
\begin{array}{ll}
SafeToStack(Obj1, Obj2) & \\
On(Obj1, Obj2) & Owner(Obj1, Fred) \\
Type(Obj1, Box) & Owner(Obj2, Louise) \\
Type(Obj2, Endtable) & Density(Obj1, 0.3) \\
Color(Obj1, Red) & Material(Obj1, Cardbd) \\
\ldots &
\end{array}
$$

- Domain Theory:

$$
\begin{array}{l}
SafeToStack(x, y) \leftarrow \neg Fragile(y) \\
SafeToStack(x, y) \leftarrow Lighter(x, y) \\
Lighter(x, y) \leftarrow Wt(x, wx) \wedge Wt(y, wy) \wedge Less(wx, wy) \\
\ldots
\end{array}
$$

SafeToStack(x,y) cont.

Determine:

- A hypothesis from H consistent with training examples and domain theory.

Prolog-EBG

Prolog-EBG(*TargetConcept*, *Examples*, *DomainTheory*)

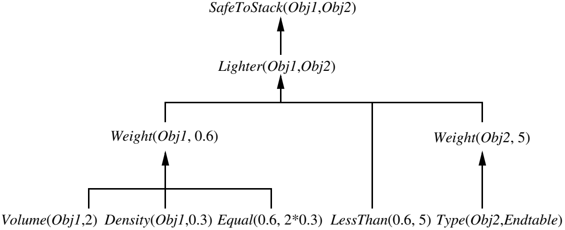
- *LearnedRules* ← {}
- *Pos* ← the positive examples from *Examples*
- for each *PositiveExample* in *Pos* that is not covered by *LearnedRules*, do
  1. *Explain:*
     - *Explanation* ← an explanation (proof) in terms of *DomainTheory* that *PositiveExample* satisfies *TargetConcept*
  2. *Analyze:*
     - *SufficientConditions* ← the most general set of features of *PositiveExample* that satisfy *TargetConcept* according to *Explanation*.
- (for each *PositiveExample* in *Pos* that is not covered by *LearnedRules*, do)
  3. *Refine:*
     - *LearnedRules* ← *LearnedRules* + *NewHornClause*,
       where *NewHornClause* is of the form

$$\mathit{TargetConcept} \leftarrow \mathit{SufficientConditions}$$

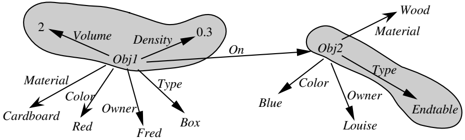
- Return *LearnedRules*

An Explanation

Explanation:



Training Example:



Summary

- In ILP sets of first-order rules can be learned directly
- Hypothesis language of Horn clauses is more expressive than feature vectors (allowing variables, representing relations)
- Suitable for learning over structured data (meshes, chemical structures, graph-representations in general)

- Not only applicable for learning classifiers but also for learning general (recursive) programs (inductive programming)
- Sequential covering algorithms learn just one rule at a time and perform many search steps

Summary

- **FOIL** is a sequential covering algorithm
    - a specific-to-general search is performed to form the result set
    - a general-to-specific search is performed to form each new rule
- Induction can be viewed as the inverse of deduction
- While Foil is a generate-and-test algorithm, approaches based on inverse resolution (**Golem**, **Progol**) are example-driven
- ILP can be naturally combined with deductive inference: Explanation-based learning allows for enriching training examples by inferences drawn from a theory.

Learning Terminology

Supervised Learning	unsupervised learning

Approaches:

Concept / Classification	Policy Learning
symbolic	statistical / neuronal network
inductive (ILP)	**analytical** (EBL)

Learning Strategy:	**learning from examples**
Data:	**structured, symbolic**
Target Values:	**concept (true/false)**